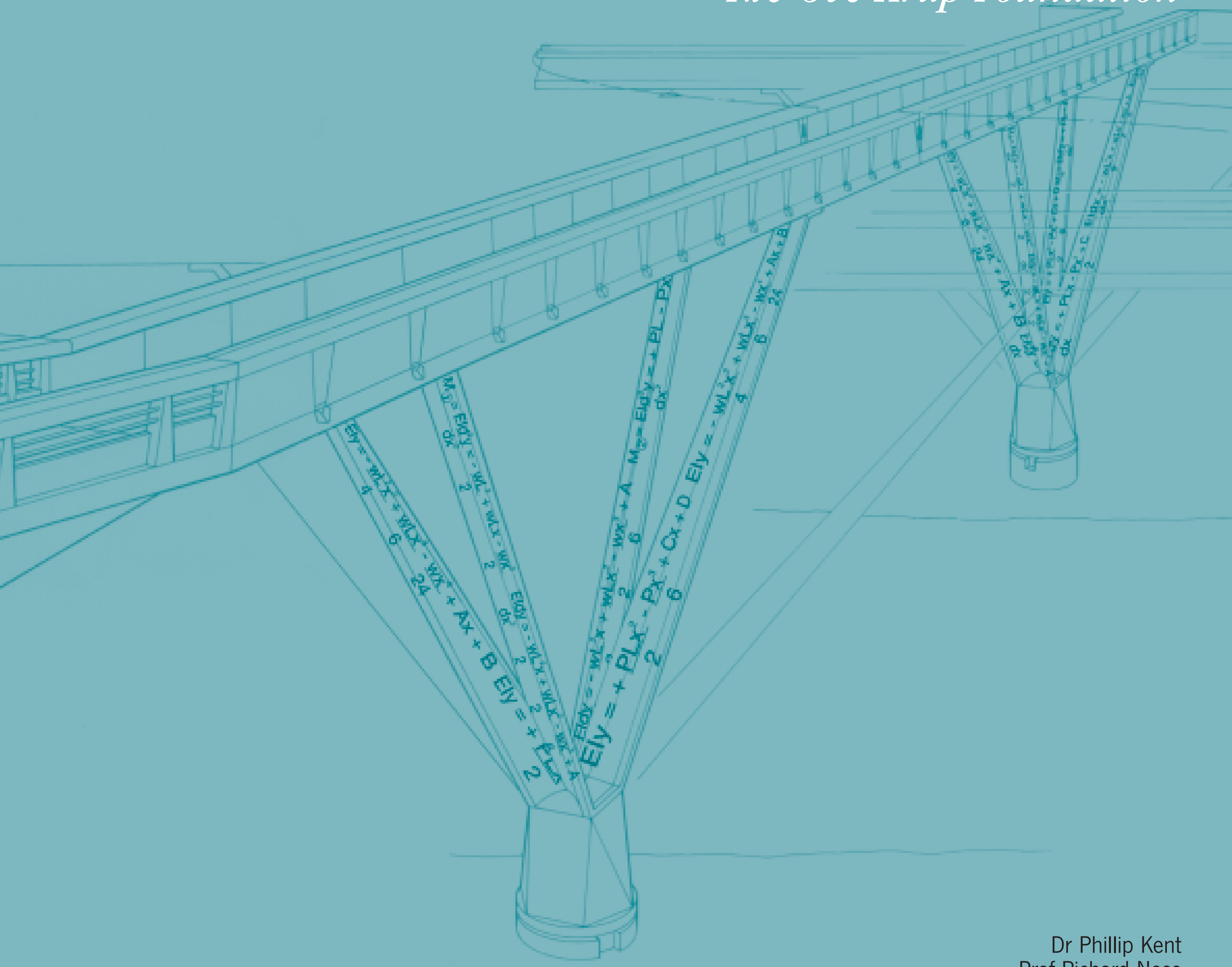
The Ove Arup Foundation

Mathematics in the University Education of Engineers

A Report to The Ove Arup Foundation



Dr Phillip Kent
Prof Richard Noss

THE OVE ARUP FOUNDATION

After Sir Ove Arup died at the age of 92 in February 1988, the Directors of Arup Group decided to commemorate his life by endowing an educational trust related specifically to the built environment. The Ove Arup Foundation was formed early in 1989, with Trustees drawn initially from Sir Ove's former partners, together with a family representative. For the Foundation's advisory committee, The Royal Academy of Art, The Royal Academy of Engineering, (then the Fellowship of Engineering), the Royal Institute of British Architects, the Chartered Institution of Building Services Engineers, the Institution of Civil Engineers, and the Institution of Structural Engineers were each asked to nominate a member. Arup Group subscribed funds to build resources that would generate about £150 000 pa in real terms. Arup Group continues to support the Foundation.

With the organising help of the Cambridge Programme for Industry, a seminar of distinguished invitees from the world of the built environment was held at Madingley Hall, Cambridge, in September 1991. Five architects and five engineers were asked to address five topics and give papers about education for the built environment. This event provided an opportunity for people who did not normally get together to discuss such issues to debate education over two days in congenial surroundings. Additionally, it brought the Foundation to the construction industry's attention, and gave the Trustees many ideas to consider. They decided on a policy of spending about 70% of the Foundation's income from capital on major self-generated schemes, with the rest used to fund external applications or to deal with smaller ventures. This policy continues, though it is subject to review from time to time.

Major Initiatives

Discussions at the Madingley Conference pointed up the need for a post-graduate course that would enable practitioners of the various disciplines to work together in a studio environment. The University of Cambridge Architecture and Engineering Departments together proposed a new venture, breaking new ground for both the University and for the industry. This was to launch a new Masters course in Interdisciplinary Design for the Built Environment (IDBE). The Foundation provided substantial funding to launch the course, and for early scholarships. It has recently agreed to provide a further contribution to enable the course to further develop, building on the reputation already enjoyed.

Ove Arup's interest in the environment, particularly in his latter years, was well known. In 1994 Mansfield College Oxford requested the Foundation's support for the Oxford Centre for the Environment, Ethics and Society. The trustees decided to sponsor a fellowship in Environmental Risk, and Maurie Cohen was appointed in autumn 1995 on a three-year term.

A third major initiative is centred at the London School of Economics, who sought support for the establishment of a new department – unique to the UK – bringing together architecture/planning, engineering, and sociology in addressing the problems of the built environment. This initiative, entitled 'City, Architecture and Engineering' launched an MSc course in 1998. The Foundation contributed substantial funds to launch the course, and continues to provide funding. The course already has wide recognition, and attracts high quality students from all over the world.

Another important initiative has stemmed from the pioneering IDBE course at Cambridge. The Foundation has provided significant funding to help launch a Masters course in interdisciplinary design and management (MIDM) at Hong Kong University. In its first year it is already oversubscribed, helping create the professional skills needed for the challenges of tomorrow's world.

The Foundation has also funded a new Chair in Creative Design in the Department of Civil & Environmental Engineering at Imperial College. This brings to a world-renowned engineering course a studio-based learning environment that stimulates the mind, and broadens the experience. There are lessons learned that can be applied elsewhere.

Alongside the sponsoring of these courses, the Foundation has commissioned a series of research papers from distinguished academics in the field of engineering education. The present report is the third in this series, following those by Prof David Gann and Dr Ammon Salter in 1999 and Prof David Nethercot and Dr David Lloyd Smith in 2001.

Donations

Some of the more significant donations made by the Foundation are:

- Cranfield Department of Applied Energy – lectureship for five years
- Partnership Awards – prize for teaching engineering for three years
- Building Experiences Trust (providing experience of the built environment for schoolchildren) – four donations
- Research by Loughborough University into 'How children choose technology' – three years
- University College London Lighting MSc – commitment for four years
- Arkwright Scholarships for students studying mathematics, science and technology – four years and further commitment
- Croydon Clocktower Project (providing hands-on construction experience for children)
- Architectural Association seminar on smart materials
- Manchester Metropolitan University – bursary for a student in Art in Architecture
- Specialist Learning Foundation – support to this initiative to improve student learning across a range of subjects in the school curriculum
- XL Wales – support to the Invention and Discovery Roadshow helping primary school students experience the excitement of maths, design and technology.

Background photograph: Sir Ove Arup inspects the works on site at Kingsgate Bridge, Durham
Front cover image: Daniel Blackhall / Sean McDermott

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The Ove Arup Foundation, London, May 2003

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We also owe a considerable debt to the many individuals in universities and industry for their willingness to be consulted for this research.

ABOUT THE AUTHORS

The authors are mathematics educators in the School of Mathematics, Science and Technology at the Institute of Education, University of London. Neither of us has any background as engineers, and we are conscious of being 'outsiders' scrutinising issues of engineering education. But we do bring experience of teaching mathematics in university (including to engineers) and schools, and have extensively researched, developed and evaluated innovations in mathematics learning in university and school, especially through the use of IT. We have also recently completed a research study of mathematics in the work of a large civil/structural engineering consultancy, and have drawn on this to support the research presented here.

Foreword

Engineering is an art where the use of imagination and deliberate choice is needed to fashion the everyday products we use, and the environment in which we all live. Engineers create wealth and wellbeing for us all, and in an ever-increasing variety of ways. And so in future we will need professional engineers with greater interdisciplinary understanding, and with more specialist skills. There will be great diversity, but all will need a deep understanding of the sciences that underpin the art of engineering, and all will therefore need the mathematical skills needed to apply these sciences.

Advances in the use of information technology and computers have transformed engineering analytical techniques, and production and management processes. These advances will surely continue, creating opportunities for all engineers, both in the way knowledge can be acquired, and then applied.

The role of mathematics in engineering education is one of these opportunities. There has been much recent debate on what mathematical skills are needed for the engineers of tomorrow, and how and when these might best be acquired. Against this background of problem and opportunity, the Trustees of The Ove Arup Foundation commissioned this study from Prof Richard Noss and Dr Phillip Kent, both mathematics educators at the Institute of Education, University of London. Their report makes interesting reading, and poses some questions that need answering. It paints a picture of a situation ripe for change, and suggests that action is needed to draw together the diverse activities and energies that already contribute to change.

The conclusions they draw are obviously seen through the eyes of mathematics educators, and not those of practising engineers. But nonetheless, they are based on research and investigation involving discussions with engineers across a broad field of academic and industrial backgrounds, and so they provide a valuable insight into opportunities for change and action. And so in publishing this report, the Trustees are taking the opportunity – in this Foreword - to draw conclusions seen through practitioners' eyes, and to call for action.

We feel it is important to focus on the following points:

- There is clear agreement that mathematical skills are essential. The questions are what, and when, and how they should be taught? These questions need answering.
- In particular, the level of mathematical skills needed at entry to engineering courses needs resolving. It is clearly connected to outputs expected from schools, but that is not the only issue. There is also the need to resolve how changes could be made to the teaching of the engineering sciences, in particular to allow students with different mathematical skills at entry to flourish. Universities, understandably, given the resources and cost involved, may well be hoping the problem will go away. In a way they are looking into a black hole that they would rather not look into, assuming entry standards that in future will not apply. Maths in the school curriculum was not a part of the remit given to the authors of this report, but it is an area of serious concern for society at large, and not just for engineers and scientists.

We are a technological society, and it is absurd that we seem to accept that we can turn out large numbers who not only lack adequate numeracy, but who also think that the subject is boring and irrelevant.

- The increasing use of IT, both in practice and in teaching needs to be seen as an opportunity. The report draws attention to the need to critically examine how IT can be used to produce ‘new symbolisms’ to explore and learn mathematical ideas, as a complement to algebraic symbolism. For example, the use of spreadsheets enables engineers to explore and communicate ideas in new ways, but supported and enhanced by an understanding of how these same ideas are derived algebraically. There is therefore added choice. The engineer has the ability to apply knowledge in new ways, selecting the right tool for the task, and also the ability to acquire that knowledge in a new way. And so although engineers will continue to need to understand how the equations that are used to solve engineering problems are derived, they will increasingly be accepting solutions to these equations calculated by computer programs created by other engineers. There is a long tradition already to this principle, with the use of tables and slide rules, where engineers accepted that the underlying principles had been properly applied. The Kent and Noss proposition is that engineers' ‘fluency’ will be enhanced by new methods, not diminished.
- Universities are exploring how to place more emphasis on studio-based or problem-based learning. In our view there are compelling reasons to do so, emphasising the creative nature of engineering, and making it more attractive to young people. But the Universities do not appear to be addressing what needs to be done to the maths syllabus as a result.
- We are much attracted by the concept of mathematics being ‘pulled’ rather than ‘pushed’ into the engineering context. It would be a natural consequence of pursuing a studio-based, design-led, approach.
- There is a particular emphasis in this report on civil engineering. We feel that much of the content is equally applicable to other branches of engineering. This presents another opportunity, because studio-based learning encourages interdisciplinarity. Debate across the disciplines should therefore be encouraged.
- There is a clearly expressed view that national leadership is needed to stimulate and spread innovation in this field. The implication is that nothing much will happen without a more interventionist approach. Such intervention need not and should not stifle innovation and change by prescribing rigid syllabi, but accrediting authorities could intervene to stimulate progress. We think they should do so. We suggest that a small and enthusiastic working group of representatives of engineering departments, mathematics specialists and practitioners should design some alternatives. The accrediting institutions must participate, or perhaps lead the exercise. And there should be young people with recent experience as part of the process.

The Trustees feel that the opportunity is greater than the problem. We hope that this report will help those who share our view seize the opportunity and promote action, including further research.

Richard Haryott, Chairman

May 2003

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The appendices to the report can be downloaded from http://www.ioe.ac.uk/rnoss/REMIT

SUMMARY

1.1 Aims and methods

This research was commissioned by The Ove Arup Foundation to survey the current roles of mathematics in undergraduate engineering education in the UK, with a particular focus on civil engineering, and to identify some visions of future directions for the teaching of mathematics. The research took place from May to December 2002, using a methodology of interviews and visits with universities, professional institutions and civil engineering companies, supported by a literature review and a questionnaire survey of university civil engineering departments.

1.2 The current debate on mathematics in civil engineering

Mathematics plays a central part in the formation of civil engineers in the UK, both in the form of an entry requirement to undergraduate courses, and as a core underpinning element of those courses. The role of mathematics in civil engineering has been a high-profile issue for some years, and it recently came to the fore when the Joint Board of Moderators decided to stimulate debate by temporarily removing the requirement for A-level mathematics as an entry qualification to civil engineering degrees.

1.3 Mathematics: Problem or opportunity?

The pressing need to recruit and retain students on engineering courses means that it is natural for academics to focus on the *mathematics problem* at the interface between school and university. Yet the role that mathematics plays in professional practice has changed radically in the last 30 years. Today, 'computational mathematics' is perceived as a tremendous *opportunity*, pushing forward the boundaries of civil engineering design. Within this, mathematics as explicit work by individual engineers has evolved into mathematics as a distributed activity across design teams and the computers that support them.

In order to resolve the apparent contradictions of mathematics as problem or opportunity, it is necessary to consider the different uses of mathematics in engineering practice: the *direct* usefulness of mathematical techniques and ideas to practice (eg to do a load case analysis of a certain kind of structure) and their *indirect* usefulness - the ways in which mathematics contributes to the development of engineering expertise and judgment. Mathematics is and will remain crucial, and this report addresses a set of questions facing engineering mathematics education:

- What types of mathematical knowledge do engineers need?
- How can the minimum level of mathematical knowledge that remains essential to engineering practice be characterised? On what does this knowledge rely?
- How does computer technology change this situation?
- When and how should mathematics be taught?

2.1 What is engineering mathematics for?

Since the widescale introduction of computers to engineering practice in the 1980s, the civil engineering profession has been in a long period of transition concerning the mathematical work done by individual engineers. Some aspects of engineering mathematics clearly remain essential whereas other aspects have changed considerably. In-between, there lies a large 'transitional area', where the boundaries between 'relevant' and 'irrelevant' are shifting, and not in the same way for all the types of work encompassed by civil engineering.

There is widespread agreement across the engineering profession about what is the desirable mathematical competence of a graduate: 'Mathematics should instil disciplined thinking and rigour in the development of arguments based on assumption and simplification in modelling, should teach the importance of controlled approximation, and, above all, impress upon students its value as a tool to be invoked when quantitative evidence is needed to underpin assertion, hypothesis, or sheer physical intuition' *(Nethercot & Lloyd-Smith)*. Whilst few would dissent from such a description, there are questions about what meanings people attach to the different terms; even: what is 'mathematics'?

Practising engineers do not tend to regard mathematics as a problem area, or something that they have they felt the need to engage with directly. Practitioners regard confidence with mathematics as crucial for the majority of engineers, and above all, they require a balance of skills across engineering teams. Different employers need different balances: even civil engineering design consultancies still only require 10-20% of their engineers to have specialist skills in analysis.

In the past, engineers had to learn a lot of mathematics for practical purposes. At the same time, they could be expected to absorb some understanding of mathematics as a 'logical way of thinking', and its importance as part of a practising engineer's expertise. The availability of the computer as a calculating tool has undone this relationship between practical and theoretical aspects. The teaching of 'practical' mathematics is becoming much more focused on the process of modelling of engineering systems - this results in a decrease in the teaching of calculation techniques, but it does not mean that all manual work with mathematics can be replaced: there is a need to find the right balance. To teach mathematics as a way of thinking, there is a traditional model in the use of formal mathematics-older engineers can look back to courses in Euclidean geometry in their schooldays. New models for this aspect of mathematics learning are not yet widely agreed. Is a 'logical' way of thinking only to be gained through studying mathematics? Could the same kinds of 'analytical problem solving' also be developed through, for example, requiring all students to develop substantial skills in computer programming?

2.2 Key issues in civil engineering education

The most pressing recent influence on civil engineering education has been the declining entry to civil engineering courses. There has been a general decline across all engineering disciplines, so the issues for civil engineering stem both from common factors (such as the changing school curriculum and examination system) and particular factors for civil engineering and construction: there are widespread skills shortages, as well as problems with professional institutions being sufficiently open to the diversity of specialist skills now encompassed by the civil engineering profession.

Engineering education in the UK is framed by the regulated structure of engineering careers (SARTOR). The shortcomings in the current version of SARTOR have engendered a major review, which is likely to shift emphasis away from controlling inputs to engineering careers (eg entry qualifications), towards an emphasis on 'output standards' for engineering degrees. This raises significant questions for mathematics - should 'mathematical competence' be an output of an engineering degree, and how should it be defined?

2.3 The 'mathematics problem' in higher education

The problem of a gap between the expectations and reality of the mathematical competence of new undergraduate students has been a concern across all numerate degree subjects since the mid-90s: students are perceived to lack fluency in algebraic manipulation, to lack analytical powers for multi-step problems, and to lack a proper appreciation of what mathematics is about in terms of the roles of precision and proof. It is now accepted that A-level no longer provides what it used to as a preparation for university study, and significant steps are under way to reform the whole school mathematics curriculum.

We looked at employer's perceptions of the importance of algebra and symbolic manipulation, and found less concern with manipulative skill *per se* than with the need for graduates to have an 'holistic' awareness of mathematics, to be able to recognise where mathematical work is going to be required in the context of engineering, and in knowing the point of referral to a person of greater analytical skill.

An area of mathematical problems, which has received much less attention than algebra, is that of geometry and spatial visualisation. This was a concern expressed by both practising and academic engineers. Formal geometry used to be an organising principle for all of school mathematics, and some have argued that it should be again. It is worth considering whether geometry could take more of an organising role for undergraduate engineering mathematics: it is ironic that mathematicians see geometric structure in all of the areas of mathematics studied by engineers (calculus, linear algebra, etc), but this is hardly taught to engineers, whose primary encounters with mathematics are strongly algebra-based.

While changes in school mathematics are under consideration, there is a danger for mathematicians and engineers in universities to seek to return school mathematics, and therefore beginning university mathematics, to 'how it used to be'. Even if this were possible, would that be the only possible mathematical preparation for a 21st century engineer? In the work of practising engineers there are now many 'symbolic fluencies' in use (ie the expression of mathematical relationships in the various symbolic forms provided by computer software) alongside traditional pen-and-paper algebraic symbolism. The possibilities of these 'new symbolisms' need to be critically examined in engineering education, especially considering the diversity of engineering courses that the engineering community is seeking to develop.

3.1 Engineering degrees and the place of mathematics

Current degree programmes fall into three types, MEng, BEng and BSc (the first two of these are CEng-accredited and the third is IEng-accredited). It is not clear at the moment, given the restructuring of degree courses and professional status that is under way, if this CEng/IEng split will remain for degree-based entry into the profession. However, there is a clear need for broader, 'less technical' courses for civil engineering - which in the form of BSc degrees (with IEng -accreditation) have not taken off-since this kind of graduate is in seriously short supply. For such students, there is a generally acknowledged need to teach mathematics in less depth but still to convey meaning and understanding. There is significant scope to reassess mathematics teaching in this context: it has been proposed that such teaching should take a much more explicitly holistic approach which seeks to establish a mastery of mathematical techniques taught through engineering contexts, and with the integrated use of mathematical software.

The debate around the development of BEng and MEng degree programmes is much more contentious than for BSc degrees. Whereas there is a broadly-accepted need for degrees to change in terms of introducing more design-oriented studies, we found considerable resistance to the idea of changing mathematical content, or mathematical entry requirements. However, given the common problems of lack of mathematical fluency that students experience, there is definitely a tension between the need to maintain 'rigour' and the difficulties for students in handling this via mathematical symbolism. The most-discussed area for this debate is structural analysis, where the analytical content of courses has been gradually reduced, and more emphasis given to the use of computer software.

3.2 IT and engineering mathematics

IT has revolutionised the use of analytical techniques in civil engineering practice. More recently, it has begun to make deep impacts on analytical courses at undergraduate level (eg in structural analysis) though the adoption of IT into these courses is still controversial as there is disquiet that shallow use of technology will replace principled understanding. In the area of mathematics courses, revolutionary possibilities have been discussed for some time, and software in the form of general-purpose computer algebra (symbolic manipulation) systems is now available to challenge the mathematics curriculum in ways similar to those that obtained in analytical engineering courses. However, engineering mathematics courses have not undergone widescale changes. Two negative factors are the high cost of moving 'chalk and talk' mathematics teaching out of lecture rooms and into computer laboratories, and the lack of a common grounding in mathematical technology in school mathematics curricula. There are significant dangers in losing the teaching of pen-and-paper mathematical techniques to 'button pressing'. It is necessary to apply a 'sieve of relevance' – to assess what fundamentals need to be retained and how they may be incorporated into an IT-oriented syllabus.

3.3 Some examples of revised approaches to mathematics teaching

This section contains four examples that illustrate the range of responses being made to problems with mathematics. In a survey across UK civil engineering departments, we generally found disquiet about the state of mathematics teaching, but curriculum changes are mainly taking the form of gradual 'curriculum shift' rather than widescale systematic change. Whilst not all engineers agree that widescale change is required, much more activity can be expected as current developments in professional regulation and school mathematics impact fully on universities.

3.4 Mechanisms for change

Engineering mathematics curricula often contain topics that are present for historical reasons, but which are no longer used in engineering courses. These glitches can easily arise where there is a lack of dialogue about mathematics teaching. Perhaps common ground can be gained by a constructive dialogue on two fronts: on the mathematical topics in the curriculum, and on delivery and pedagogical approaches. These two issues are intertwined, and consideration of one without the other leads inevitably to misunderstanding and inertia. Engineering courses are tending towards 'design-based' approaches (employing design as an organising principle), with a decreasing use of 'chalk and talk' pedagogy, and mathematics courses will have to accommodate themselves to this trend (this does not necessarily entail a radical departure from existing mathematics teaching). Although a number of methodologies are being chosen for design-based learning, the effect on mathematics is similar: the need for analysis is 'pull, not push' - the need can emerge where design requires it, not pushed into the student prior to having a meaningful context for it. In fact, some mathematicians are already advocating this kind of change. In the past, a valid objection to pull-based mathematics has been the uncomfortable notion of using a mathematical idea before knowing the techniques of its application. Yet carefully-designed IT use can make it possible - perhaps even desirable - to use mathematical ideas *before* understanding the techniques.

The professional institutions, ICE and IStructE, and the JBM have maintained a policy of not being prescriptive about mathematics curriculum in assessing degree programmes for accreditation. Given the extent of the problems with mathematics, and the significant costs of innovation in curriculum and modes of delivery, there is a case for the institutions to be more interventionist, both to push for change through the accreditation process, and actively to support curriculum change.

4. Conclusions

1 We have found agreement from every quarter that undergraduate engineering students continue to need to know and to learn mathematics. The fundamental question is *what kind* of mathematics is needed, and *when*.

2 The system of mathematical education in engineering formation is ripe for change –regulatory frameworks, entry routes to the profession and school mathematics provision are all likely to experience major changes in the near future. Therefore there is a need to consider the *mathematical knowledge* that is required, by whom, and in what form. For example, geometry is a key area of knowledge for civil engineers that is currently under-taught in schools and universities, and there is reason to consider making geometry more of an organising theme for mathematics courses than is currently the norm.

3 There are possibilities in the 'new symbolisms' that practising engineers use, through software, to engage with mathematical ideas. These need to be critically examined in engineering education, alongside well-understood algebraic symbolism.

4 It is time to reconsider pedagogical approaches that can best 'deliver' the mathematical needs of students. Mathematics could benefit from being more 'pulled' into the context of design-oriented engineering teaching, rather than 'pushed' into students in the absence of a context. This entails a shift in approach from teaching mathematical techniques towards teaching through modelling and problem-solving.

5 Carefully-designed IT use can make it possible to *use* mathematical ideas before *understanding* the techniques. In the pre-computational era, a strong objection to such pull-based mathematics was that to *use* a mathematical idea properly required a detailed *understanding* of the techniques of its application. But times, and technologies, change.

6 There is a need for national leadership to stimulate, and to promote the spread of, the innovative work in curriculum design and delivery currently being carried out by enthusiastic individuals and individual departments. This is a role that the professional institutions (together with cross-sector engineering organisations such as the Engineering Technology Board) should be well placed to assume, as campaigners for the engineering profession and controllers of the accreditation mechanisms for engineering degrees.

1

Introduction

1.1

Aims and methods

This research was commissioned by The Ove Arup Foundation, with the following aims:

- to survey the current roles of mathematics in undergraduate engineering education in the UK, with a particular focus on civil engineering;
- to identify some visions of future directions for the teaching of mathematics in engineering education, including a consideration of the kinds of mathematical knowledge and skills that will need to be taught in university engineering courses in the future;
- to take account of the range of possibilities of IT-based teaching and learning methods.

This report is addressed to a wide audience:

- engineers in universities and professional practice
- mathematicians and mathematics educators in universities
- policymakers, teachers and all stakeholders in mathematics and engineering education and practice.

We are reporting here on mathematics as used by civil engineers, so whilst some of what we will say does extend to other engineering disciplines, it is evident that some of it will not. In particular, other disciplines (especially electrical/electronic) use abstract mathematical techniques more heavily than does civil. Conversely, 'intuitive' design often plays a more important role in civil engineering practice than in other disciplines.

Methods

This study took place from May to December 2002. The methodology we adopted was a combination of:

- literature review (both in print and on the internet)
- interviews (by way of email, telephone and site visit) with universities and civil engineering industry; also engineers from other disciplines, and mathematicians involved in teaching engineering mathematics (a list of the main contributors is given in Table 1)
- a questionnaire survey of UK civil engineering (or general engineering including civil) departments accredited by the Joint Board of Moderators. The aim of this questionnaire was to get a broad picture of courses being offered, and to use the results qualitatively rather than quantitatively, to identify interesting places and initiatives for further investigation. *See the report website for the questionnaire text and the analysis of responses.*

An original aim of this research, which we were unable to pursue to the level of detail intended, was to develop some exemplars, or 'fragments of new mathematics curriculum', especially to examine the role of software in engineering mathematics teaching. This would be a valuable exercise for future research, and the examples given in Section 3.3 do suggest some possible starting points for this.

We have focused on the entry route to the engineering profession via A-level study and undergraduate engineering degrees. Of course there are other routes towards engineering degrees, and non-degree qualifications *(cf Engineering Council 2001)*. In terms of mathematics, the alternative post-16 qualifications (principally Advanced GNVQ) have had low take-up amongst students applying for engineering degrees, and there have been concerns about how well these courses prepare students in traditional 'basic' mathematics skills *(see Engineering Council 2000, James 2001, Mustoe 2001)*.

We do not report here in any detail on the current reviews (by the DfES, QCA, etc) of the school mathematics curriculum and examination system. Whilst these can be expected to have considerable impact on undergraduate engineering mathematics in the next five to 10 years, it fell outside our remit to consider these any more than tangentially.

We have sought feedback on our research findings in several ways. Part-way through the period of the research, we prepared a brief overview document of the research results, and this was distributed to the respondents we had contacted via the interviews and questionnaire with an invitation for feedback. We also presented provisional findings of the research at several seminars and conferences. In addition, this report has been informed by our own previous research and teaching experience in school, university and workplaces, most recently a public-funded research project in 2001, 'The Mathematical Components of Engineering Expertise', which made an observational study of mathematics in the work of a large civil/structural engineering consultancy *(Kent & Noss 2002a, 2002b)*.

Table 1: Organisations consulted

Universities	LTSN Centre for Engineering
Birmingham	LTSN Mathematics Statistics & OR Network
Bristol	
Heriot-Watt	**Institutions**
Imperial College London	Engineering Council (UK)
Leeds	Institution of Civil Engineers
Loughborough	Institution of Structural Engineers
Manchester	
Nottingham Trent	**Industry**
Portsmouth	Carillion plc
Southampton	Arup
Staffordshire	Peter Brett Associates
UMIST	Whitby & Bird
University College London	

1.2 The current debate on mathematics in civil engineering

Mathematics plays a central part in the formation of civil engineers in the UK, both in the form of an entry requirement to undergraduate courses, and as a core underpinning element of those courses. Civil engineering undergraduate courses have experienced a decline of 50% in student numbers since the mid-1990s, and this has been more severe than in other engineering disciplines (see Figure 1). Mathematics is strongly implicated in this trend, since, among other things, students have required A-level mathematics to enter a BEng/MEng course; school students are being turned away from studying mathematics (less than 10% of students elect to do a full A-level in mathematics), and all physical science, mathematics and engineering departments have found themselves fishing in this declining pool of candidates.

The role of mathematics in civil engineering formation has been a high-profile issue for some years, but it has recently come to the fore because of the modification in the regulations of the Joint Board of Moderators (announced in May 2002), to allow students without A-level mathematics to enter undergraduate degree programmes in civil and structural engineering: the Board 'has decided not to insist that A-level Mathematics (or equivalent) is a prerequisite for entry to a JBM accredited course', provided that students' advanced mathematical skills are developed within the course [personal communication with Professor James Croll,chair of the Joint Board of Moderators]. This is a temporary measure for two years, adopted especially in the light of the 'Curriculum 2000 problem', which saw students taking AS Mathematics exams in the summer of 2001 experiencing very high failure rates (30%), with a large consequential drop out for progression to the second year of mathematics study *(Nicholson & Belsom 2002)*.

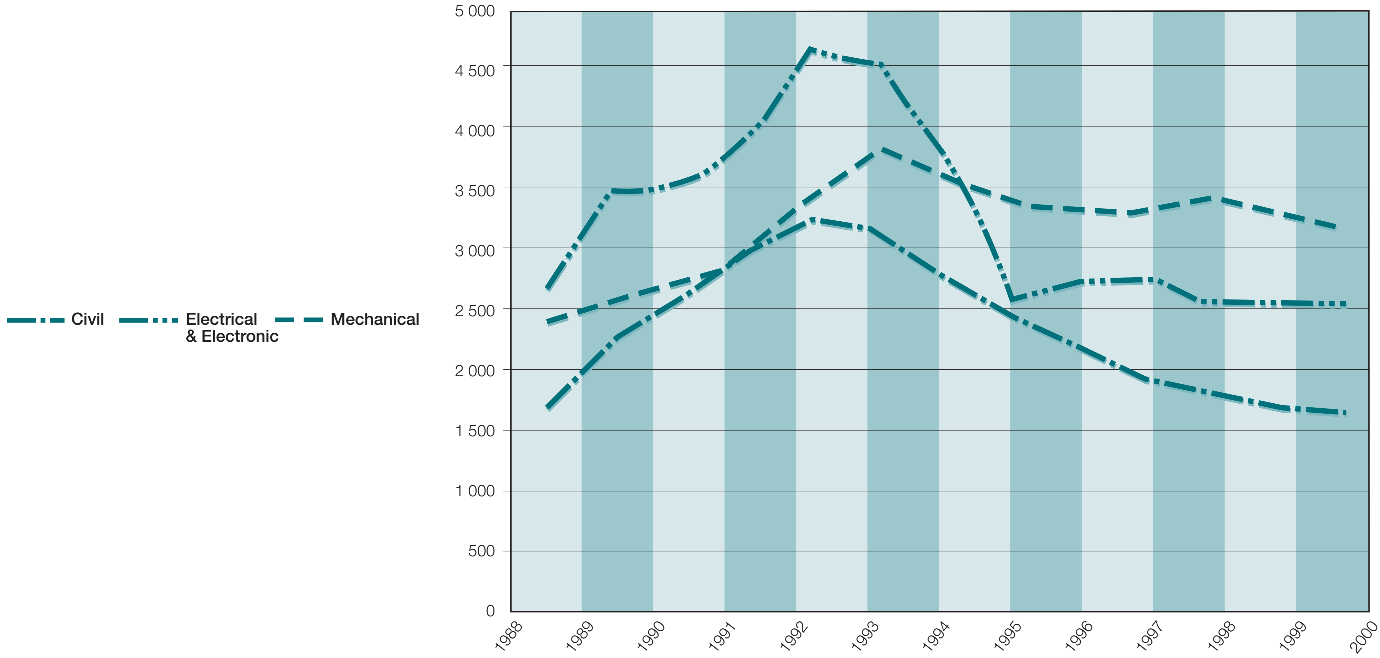
The expectation was that civil engineering departments would be receiving significantly fewer applications for at least the next one or two years. Whilst the drop-out has (not surprisingly) impacted severely on recruitment to mathematics degrees this year, applications to physics and engineering courses have actually risen - by 2% in the case of civil engineering, a trend partly attributed to a major decline in applications for computer science and other IT-oriented degrees *(New Civil Engineer, 3/10/2002)*.

The current debate in mathematics takes place in the light of a longer-term trend, a subject about which the JBM's regulation change has also sought to stimulate discussion. There is a feeling expressed by some practitioners and academics that the A-level mathematics requirement is acting as a block against allowing a more diverse group of people to enter into the civil and structural engineering profession [1] :

Many universities … are looking towards providing education programmes through which other talented individuals could be attracted into civil engineering. Such degree programmes could develop the necessary technical and mathematical knowledge and understanding within the programmes. (Croll, personal communication)

In other words, if civil engineering firms are going to be able to become the leaders in the construction business, taking the initiatives on the politics and financing of projects, there is a need for a more diverse range of engineers in practice. However, the need for a core of 'mathematical' engineers remains, and, according to Croll, there will be a 'development of degree programmes with even greater mathematics and engineering science content' than is currently the case. This is the first instance of a major theme of this report: the apparent divergence of opinion about 'what maths do engineers need?' is in fact tied up with a more complex question of 'what kinds of engineers does the profession need?'

Figure 1: Number of students entering engineering courses, 1988 - 2000
[Data source: Engineering Council (2001)]



The JBM's announcement prompted a good deal of discussion, for example in the civil profession's weekly journal, *New Civil Engineer ('Does dropping A-level maths add up?', 14/11/2002)*. A good representation of the range of opinions is provided by an earlier 'debate' column in *New Civil Engineer (13/06/2002)*. The following question was posed: Is A-level mathematics an essential grounding for a future civil engineering professional[1]?

Replying 'Yes', Professor Muir-Wood of Bristol University:

Some 72% of UK universities run civil engineering courses that are not viable; 27% of civil engineering graduates do not join the industry. Admissions tutors know there are too many courses chasing too few students with the necessary combination of entry qualifications and commitment. One response is to lower the entry hurdle, removing the need for civil engineering undergraduates to have A-level mathematics.

To which he counters:

Perhaps we should give fewer students a broader civil engineering education developing both the mathematical skills and the creativity that enables civil engineers to work confidently with other construction professionals. … Professional engineers need to communicate mathematically,

[1] Another rather important diversity and recruitment problem which we will not discuss here is the gender imbalance amongst engineering students – only about 15% of entrants to civil engineering courses are women, and this figure has not changed significantly for many years *(cf. Engineering Council 2001)*.

verbally and graphically. Without a facility in mathematics, engineers are cut off from the logic of scientific discovery.

Replying 'No', Jon Prichard, professional development director at the Institution of Civil Engineers (ICE):

Civil engineering is competing against an increasing spectrum of careers requiring numeracy skills. The pool of those obtaining such skills is getting smaller … the JBM had a choice -to maintain the status quo and contribute to the deepening skills shortage; or to remove temporarily a self-imposed obstacle to course admission … An inflexible approach would have commenced a downward spiral, probably leading to further closures of civil engineering departments. Instead, the JBM has taken the pragmatic view that those running civil engineering programmes can elect to change the emphasis of their entry requirements without diminishing their learning outcomes. There are many talented and bright individuals who do not currently sit maths A-level, who can now take up a challenging and rewarding career in our profession. …. As we take the next steps into the digital and multi-disciplinary team-working era there is scope for others with talents that are less directly grounded in maths to join us. If a relaxation of the maths entry requirements is successful, then perhaps we should consider permanent adoption. We shouldn't be afraid of change.

1.3 Mathematics: Problems or opportunity?

How can we make sense of this apparent divergence of opinion? It seems that the two writers are in fact answering different questions, and perhaps considering different notions of the idea of a 'civil engineering professional': one is defending the existing 'CEng' degree programme, and the kind of chartered engineers which emerge from it, the other is arguing for diversity in the education system to support a diverse profession, employing a range of types of engineers. Implicit in this is a difference of view about what a professional engineer is: Muir-Wood talks about one role of engineers in practice; Prichard, again, argues for other roles. Thus, the debate on mathematics is locked into a large, complex debate concerning the structure of the civil engineering profession as a whole ('what is a civil engineer?').

We have heard from a range of voices in civil and structural engineering, across academia and professional practice, and it is clear to us that there is no unanimity of feeling or opinion, and certainly not unanimity in responses to the problems and issues that are affecting undergraduate education. A useful way to make an overview of the feelings about mathematics is to consider the language used to talk about it.

In the practice of civil engineering, 'computational mathematics' is perceived as a tremendous opportunity, pushing forward the boundaries of civil engineering design. For example, in the structural design of buildings, there are high-profile, high-budget projects (eg the British Museum Great Court) pioneering the development of techniques that gradually become used across the whole range of building projects (in the case of the Great Court, computer-based 'form finding' and structural analysis, combined with CAD/CAM automation of the fabrication).

In contrast, in universities, mathematics is often described as a problem looming over engineering education. For example:

Each year the A-level results come out showing increased pass rates, yet we do not see any improvement in the ability of students to tackle the mathematics of engineering degree courses. … The situation is serious, and getting more so. Most university engineering departments now find it necessary to provide remedial teaching for students whose mathematical foundations are not adequate for university first-year maths. (Pyle 2001)

Under the present circumstances, it is natural for academics to focus on the 'mathematics problem' at the interface between school and university - because of the pressing need to recruit students into engineering courses, and to retain them (see *Cutler & Pulko 2001* for a recent collection of papers on the 'retention problem'). The common complaint is that students, sometimes even those with a good grade in A-level Mathematics, lack fluency and 'comfortableness' with mathematical symbolism and its manipulation, and this compromises the whole development of engineering understanding from a mathematical basis.

From the perspective of professional practice, the 'mathematics problem' can be viewed somewhat differently. We have heard opinions that academics might fruitfully consider the place of mathematics at the other 'interface', between university and industry. The role that mathematics plays in practice has undergone radical changes in the last 30 years, in terms of mathematics as explicit work by individual engineers becoming mathematics as a distributed (and more implicit) activity across design teams and the computers which support them. This argument is often expressed in terms of the usefulness (or not) of knowing mathematics in engineering practice. Posed in this way, it is not surprising that the current debate has often become rather polarised. For example the JBM's announcement on A-level Mathematics was interpreted by some engineers as a proposal that 'mathematics is not necessary for civil engineering', so that the members of the JBM have had to spend some considerable effort correcting this impression: 'We have not dropped the requirement for maths, just when you get it' *(New Civil Engineer, 14/11/2002).*

There is a complexity in the different uses of mathematics in engineering practice: not only the *direct* usefulness of mathematical techniques and ideas to practice (eg to do a load case analysis of a certain kind of structure), but the *indirect* usefulness of mathematics to practice, that is, its *formative role in the development of an engineer* - the ways in which mathematics contributes to the development of engineering expertise and judgement. For example, matrix algebra plays a role in developing an understanding of many engineering principles, but matrices in explicit form may be of little practical 'use' to a working engineer. It's easy to see that the direct use of mathematics in practice has changed, but the changes in the indirect uses are less apparent.

Where academic engineers defend the place of mathematics as it currently is taught, they are, at least in part, defending its formative role - how it enables students to gain an engineering understanding even though it may subsequently 'fade into the background' of the graduate engineer's thinking. But it surely needs to be debated: is the way of teaching mathematics at present still optimal? More than one practising engineer expressed the view (however unfairly) that academics tend to be conservative, and given current academic workloads, may be unwilling to consider wide-scale changes to mathematics teaching.

This report addresses a set of questions facing engineering mathematics education at the present time:

- What types of mathematical knowledge do engineers need?
- How can the minimum level of mathematical knowledge that remains essential to engineering practice be characterised?
- On what does this knowledge rely?
- How does computer technology change this situation?
- When and how should mathematics be taught?

Mathematics is both a problem ***and*** *opportunity for universities. The question is how to educate students mathematically to master mathematics-based technologies?*

2

Mathematics in engineering education: The current situation

2.1

What is engineering mathematics for?

Mathematics in transition

The civil engineering profession is in a long period of transition concerning the work that mathematics does for engineering, and the mathematical work done by individual engineers. The present leaders of the profession, in their 50s and 60s, experienced their educational formation in a time when computers were exotic and specialist tools, and computers were not common in engineering practice or education until the appearance of cheap microcomputers in the 1980s. Nowadays, the computer is ubiquitous in the working lives of young civil engineers; this has led to some questioning of the value of (some) pen-and-paper analytical techniques:

Who in practice nowadays would conduct an elastic analysis of a single bay pitched roof portal frame other than by feeding it into the office program? Yet university libraries contain shelves of structural textbooks with pages devoted to complex and impenetrable hand methods for analysing such structures. Of course, students need to appreciate the difference between response under vertical loading and sway and to have some way of checking that the computer-generated answers are reasonable … They do not, however, need to master archaic techniques for conducting the analysis with pencil and paper when such methods are never used in the 'real world'. (Nethercot 2000)

Some aspects of engineering mathematics clearly remain crucial: the possession of a mental sense of 'numbers', the ability to approximate scales and orders of magnitude, the ability to perform approximate mental calculations, the 'appreciation' of engineering principles based on mathematical ideas – and how all these contribute to professional engineering judgement. Other aspects have obviously changed; in structural analysis, for example, the zoo of specialised hand techniques for structural analysis of particular structural forms have all but disappeared from practice. In-between, there lies a large 'transitional area', where the boundaries between 'relevant' and 'irrelevant' are shifting, and not in the same way for all the types of work encompassed by civil engineering.

Furthermore, different kinds of mathematics may be relevant at different times in an engineering career. Take the case of calculus: it is essential during formal engineering education to appreciate basic engineering principles, whilst in practice it may rarely feature explicitly for many engineers – yet it is still present, 'half remembered' as part of the engineer's analytical appreciation.

In speaking to practising engineers, we have been struck by their appreciation of the power of mathematics and at the same time the limits of its applicability. Engineers use mathematics as a tool to model reality, to calculate, predict and design. Analytical models have different degrees of reliability and usefulness, which are associated with a 'cost' of analysis/calculation that has to be balanced against design priorities. The mathematical models one ends up with may be potentially more precise, but too expensive to take time to solve, or be so complex as to risk obscuring crucial engineering details. Some interviewees in engineering practice were critical of graduates' appreciation of this point, and their ability to approximate and simplify:

If you believe the computer, which young graduates tend to do, then you believe that things are so complex they can only be done by computer. But for our generation, we were brought up to believe that everything can be calculated by hand, because in the early days of computers, it was more complicated to try to use a computer than to do a hand calculation, and so you had to start by making a rough calculation. Young grads nowadays do believe computer output and they can't challenge it against an intuitive feel. They do grow out of it, but it would be better if they came in to the real world with a balanced view already. They've been so long in formal education that there is a big culture shock coming into a business environment where people have completely different agendas from the joy of mathematics and structures.

It seems to me that in the area of mathematics, the links between academic work and practice have not even started to be made. Why should this calculation have some meaning in a particular context? (Structural engineering practice)

Our perception is that there is overall agreement across the engineering profession (academic and practitioner) about what is the desirable mathematical competence of a graduate. This has been well put as follows:

Mathematics should be seen as a medium for communicating concepts, ideas, and information in a parallel way to text – and not as the mastery of a series of abstract processes with little or no linkage to physical understanding and application. It should instil disciplined thinking and rigour in the development of arguments based on assumption and simplification in modelling, should teach the importance of controlled approximation, and, above all, impress upon students its value as a tool to be invoked when quantitative evidence is needed to underpin assertion, hypothesis, or sheer physical intuition. (Nethercot & Lloyd-Smith 2001)

Whilst no-one dissents from such a definition, there are questions about what meanings people attach to the different terms; even, what is 'mathematics'? Many practising engineers, it seems, have a dominant image of mathematics as numerical manipulation, whereas many academics may have a dominant image of mathematics as an activity with symbolic expressions.

Practising engineers' views on mathematics

The practising engineers we interviewed did not on the whole regard mathematics as a problem area, or something that they feel the need to *directly* engage with. Opinions expressed to us varied from the outspoken ('why do universities still teach all that stuff?') to the somewhat disengaged ('whatever universities have to do to get their students through the course'). Of course, they do care that graduates know their engineering, and the need for mathematics is certainly implicit in that, although it has not up to now been an issue:

*The quality of graduates does go up and down over the years, but in the main we do get what we want. I don't think we've ever thought of the variations being a problem with mathematics.
(Large building contractor)*

*We need numerate graduates and, on the whole, we are not seeing problems in that area. We are however experiencing problems with literacy in terms of verbal and especially written communication. By 'numeracy' I mean addition and multiplication, the very basic maths which engineers must be strong in. Going on from there depends on the discipline they're working in. With the use of computers much of the mathematical understanding is not as essential as it used to be when most calculations were undertaken by hand. Nevertheless in order to be sure you've got sensible answers, you need to understand what's going on. My definition of numeracy, then, is numeracy in the round: we want engineers who are able to use design software, understand the algorithms in that software to ensure we get sensible answers, and sensible interpretations of those answers.
(Medium-sized civil engineering consultancy)*

There are however some contrasting views to this. For example, *Clark (2000)* quotes the chief structural engineer on the London Eye project: 'the [design] problems which had been addressed from first principles … [and concern] that the current generation of graduates would not have the technical base to address these problems'.

Our perception of civil engineering practice is that confidence at a certain basic level of mathematics is the most important thing for the majority of engineers:

*For 95% of the work we do, the mathematics is basic, but you've got to have confidence in it. People have to be able to argue on a mathematical basis, not that the level is that high, I mean, many of us do not use calculus, the basics are multiplication and division, and an understanding of statistics.
(Structural engineering consultancy)*

Above all, employers are looking for a balance of skills across engineering teams - people with strong analytical skills are crucial, but different employers need different combinations: building contractors need rather few of them (it may be more cost-effective to contract out unusually complex analysis to a specialist design consultancy, or an expert academic), whereas civil engineering design consultancies need more, but still only perhaps 10 - 20% need to have specialist skills in analysis. It was interesting to note also that some companies tend to fill their graduate vacancies by having a preferred list of universities where they can expect to find the different 'products' that they need – the solid middle-ranking engineer, the high-flier with leadership potential, the future specialists in different technical areas. Importantly, the level of mathematical/technical competence seems to be assumed from the track record of each university course:

Our recruitment process is not that sophisticated as to worry about how much mathematics graduates have, and is mainly history-driven. We know we need a number of people with civil degrees, and we know that a certain number of universities produce a type of product that works for us. … The assumption is that a degree course has given a graduate enough 'education' to satisfy our needs. (Large building contractor)

Engineering as knowledge and engineering as process

New attitudes to 'knowledge' are developing in civil engineering education. More and more knowledge is becoming accessible and potentially relevant to the practising engineer–not just mathematical or scientific, but concerned with materials, construction techniques, design methodology, project finance, construction law, etc. There is quite broad agreement that the engineering curriculum is overloaded with knowledge and that the way forward is a shift in emphasis from teaching focused on knowledge (ie factual topics of civil engineering theory and science) toward teaching about the process of engineering (engineering design and professional practice). *Clark (2000)* describes the 'knowledge explosion':

Technical knowledge has expanded, and is expanding, at a great rate. … In the nineteenth century it would have been feasible for all of the needed technology for a professional engineer to have been covered in a conventional engineering course. … Now it is impossible even for a sub-discipline to cover all of the needed technology. This leads to the question: what should we teach? It appears that industry would like both breadth and depth. This, of course, is impossible, unless we accept breadth in some areas and depth in others. … The [construction] industry and society's requirements of it are changing rapidly and will continue to do so. Hence, there is a need to educate rather than train at university so that graduates will in the future be able to undertake tasks for which they were not originally trained. (Clark 2000)

There is a growing consensus (supported by changes in the regulation of engineering education) for *design* as an organising and motivating principle of a civil engineering degree. We consider in Part 3 the implications for mathematics because of this.

The traditional combined role of mathematics in engineering education

In the past, engineers had to learn a lot of mathematics for practical purposes. At the same time, they could be expected to absorb some more 'meta-level' knowledge about mathematics as a 'logical way of thinking', and the importance of that way of thinking as part of a practising engineer's expertise, alongside practical experience, physical knowledge and intuitions, codified knowledge of the profession, etc. In extreme form this view presents itself as 'mathematics is good medicine' (or 'intellectual muscle') - the view that it does not matter what particular mathematical topics one studies, but to learn the way of thinking.

The availability of the computer as a calculating tool has undone this relationship between practical and theoretical aspects. Both still matter, and of course they remain inter-related, but it is possible to consider the two separately.

Mathematics for practical purposes: a toolkit of analytical techniques and a language of communication

No longer do we have to 'plough through' long pages of deductive proof – the computer will do it for us. No longer do we have to grind through long calculations – the computer will do it for us. The challenge has changed from the ability to do this to the ability to interpret the meaning of mathematics to engineering and herein lies the challenge and change of emphasis. (Blockley & Woodman 2002)

The orientation on modelling that is taking place in engineering courses, in the context of design, and the corresponding decrease in the techniques of calculation does not mean that all manual work with mathematics could be replaced - clearly there is a need to find the right balance. Even the most devoted advocates of a change of emphasis will readily testify that some pen-and-paper techniques remain essential in the context of checking computer calculations, and also to appreciate the principles underlying mathematical models.

The need for engineers to use mathematics as a means of communication has not decreased as a result of the introduction of computer technology, however mathematical communication now takes on more diverse forms - a rough calculation worked out in an Excel spreadsheet is as common as a pen-and-paper calculation.

Mathematics as a way of thinking: a means of teaching 'logic' (precise argument)

All structural engineers should experience a rigorous mathematical education, not necessarily because they will use the mathematics in their future careers, but because of the mode of thinking that such education develops.(Clark 2000, p. 149)

There is a traditional model for achieving this, in the use of formal mathematics; older engineers can look back to courses in Euclidean geometry in their schooldays. At the moment, new models for this aspect of mathematics learning are not widely agreed upon. It may well be difficult to address this at the same time and in the same place as working with modelling. One proposal suggested to us was that mathematicians should be asked to give 'inspirational' lectures to engineering students - not given with any practical intent, but to convey the intellectual nature of mathematics as a subject. However, mathematics is a subject that has to be done to be learnt, and that calls for the need to spend time on it, and very probably to assess it.

It is also reasonable to consider whether the 'logical' way of thinking, as required by engineers, can only be gained through studying mathematics - whether the same kinds of 'analytical problem solving' could also be developed through, for example, major student projects in computer programming? The use of mathematical software (eg Maple, Mathcad) might provide just such an opportunity (provided it is remembered that software in itself does not do anything, rather it is how it is combined with an appropriate curriculum and teaching style).

2.2

Key issues in civil engineering education

Recruitment problems for engineering courses

The most pressing influence on civil engineering education over recent years has been the declining entry to civil engineering courses, as illustrated in Figure 1. In fact, the particular problem of recruitment is not at above-average A-level grades (20+points), but the middle ranking students of 15 or so A-level points. It should be noted that this has been a general decline across all engineering disciplines, so the issues for civil engineering stem both from common factors (perceptions of engineering amongst the public, and amongst students/teachers in schools, the changing school curriculum and examination system) and particular factors for civil engineering and the construction industry–where there are widespread recruitment problems and skills shortages *(see ICE 2001, Engineering Council 2001, Gann & Salter 1999)*, as well as problems with the professional institutions being sufficiently open to the diversity of specialist skills now encompassed by the civil engineering profession *(Fleming 2000)*.

Most university civil engineering departments have considerably fewer students (up to 50%) than eight or nine years ago. Some universities have taken the decision to close certain courses completely (as has also been happening to mathematics and physics departments), whilst others are recruiting increasing proportions of non-UK students (eg Portsmouth University is now 50% non-UK students in its BEng/MEng degrees, compared with 100% UK 15 years ago); the British CEng engineering degree is highly-respected internationally, and is attractive to overseas students. At the same time, an opposing trend is that engineering degrees are expensive to run, compared with the social science and humanities degrees that have been more in fashion with UK students, and this funding difference is attracting the scrutiny of university vice-chancellors.

Engineering education and professional registration

Engineering education in the UK is essentially framed by the regulated structure of engineering careers, since these regulations prescribe in many ways the nature of undergraduate education, and also currently impose entry requirements on students in terms of A-level scores. This framing is particularly an issue for civil engineering, compared with other disciplines, because there is a strong connection between career progression in the construction industry and attaining recognised professional status. In contrast, the electronic engineering industry, for example, relies very much less on professional status - it has been rapidly revolutionised by digital technologies and is now an extremely diverse field, requiring specialist expertise spanning across to disciplines outside the conventional boundaries of engineering.

Professional engineering education, and the subsequent achievement of professional recognition in the UK is governed by a set of regulations called SARTOR, Standards and Routes to Registration *(Engineering Council 1997)*. A major review of SARTOR is currently in progress, and below we describe some of the likely changes and how this will impact on mathematics education for engineers. Currently, SARTOR covers three types of professional registration: Chartered Engineer (CEng), Incorporated Engineer (IEng) and Engineering Technician (EngTech); this report is concerned with the first two of these. SARTOR is managed by The Engineering Council (UK), the national registration body for engineers in all disciplines. However, an engineer applies for registered status by becoming a member of a particular professional Institution, and two of these relevant to this report: the Institution of Civil Engineers (ICE) and the Institution of Structural Engineers (IStructE).

It is important to note that formal engineering education is considered to be the first part of a rather longer formation process. After completion of a degree, five or more years of initial graduate experience are intended to provide the practical development of 'formal' knowledge learnt at university. Under SARTOR, the completion of a particular degree course is considered to be satisfactory evidence of possessing the 'educational base' required for professional status. Such degree courses must be 'accredited' by the relevant professional institutions, and in the case of civil and structural engineering, the two institutions delegate the task of accreditation work to a single Joint Board of Moderators (JBM) *(cf Nethercot 1999)*.

SARTOR imposes minimum *entry requirements* for engineering courses: 24 A-level points (or equivalent) [2] for MEng degrees, 18 points for BEng degrees, and 10 points for BSc degrees (leading to IEng registration); this requirement has been phased in over a number of years, and currently at least 80% of the student intake to a course must satisfy the points rule. A four-year MEng degree is the educational base required for CEng qualification; an alternative to this is a three-year BEng degree plus an additional 'matching section' of one year of educational study (full-time, or part-time equivalent whilst employed). The BSc/IEng route was created in the latest (1997) version of SARTOR, and was promoted as one that should develop considerably, aiming to appeal to students with a 'less academic' interest in engineering. At the time of publication, the Engineering Council quoted an estimate of ultimately reaching a state of 70% of all engineers qualified as IEng and 30% as CEng; in fact, the outcome has been very different–in civil engineering, only about 5% of the registration applications made to the ICE each year are for IEng registration.

Mathematics and SARTOR

SARTOR does not require A-level Mathematics as part of the entry standard for degree course. Instead the specification is in terms of what the degree course must provide:

To obtain full accreditation for CEng, it must be possible to show how MEng graduates could achieve … An understanding of mathematics as a method of communicating results, concepts, and ideas … The ability to use mathematics as a tool for solving complex problems…' (Engineering Council 1997: Part 2, section 4.1.1, para 14)

A-level Mathematics used to be specified, however, in the requirements of the JBM for civil and structural engineering: 'Mathematics must be included at A or A/S level as a required entry qualification. A science subject is preferred in addition to mathematics' [3] ('JBM Accreditation Guidelines, Entry qualification for accredited degree courses', 1998) [4]. The JBM guidelines still require mathematics teaching for at least the first two years of BEng/MEng courses.

[2] That is, according to the old-style grading system (A-level grade A = 10 points, grade B = eight points, etc). This has now been replaced by a new 'UCAS tariff', to take account of the Curriculum 2000 for A and AS levels, and other post-16 qualifications. [See http://www.ucas.ac.uk]

[3] The 'A/S level' specified here is that which existed before Curriculum 2000 (Advanced Supplementary) and which formed the basis for the current AS (Advanced Subsidiary) level. We are not aware of any department specifying other than a full A-level for BEng and MEng entrants.

[4] Note: Scottish universities have a rather different position with respect to mathematics because they have an extra year on courses (ie four-year minimum BEng, five-year minimum MEng) where the first year covers a lot of the material that is studied during second year of A-level elsewhere in the UK — thus students with A-level backgrounds normally enter Scottish courses at year 2. This means that effectively the JBM's A-level mathematics requirement has never applied in Scotland.

International comparison: USA

Entry to university in the USA is mainly based on a standard national test, the SAT (Scholastic Assessment Test) which consist of two parts, Verbal reasoning and Mathematical reasoning. 800 points are available on each part, and university engineering schools typically require a minimum of 1300 points. The BEng is a four-year degree, with the first year being 'preparatory'. Mathematics teaching is always outsourced to mathematics departments. The national accreditation board, ABET (Accreditation Board for Engineering and Technology) has the following criteria for civil engineering:

The program must demonstrate that graduates have: proficiency in mathematics through differential equations; probability and statistics; calculus-based physics; and general chemistry; proficiency in a minimum of four recognized major civil engineering areas; the ability to conduct laboratory experiments and to critically analyse and interpret data in more than one of the recognized major civil engineering areas; the ability to perform civil engineering design by means of design experiences integrated throughout the professional component of the curriculum; an understanding of professional practice issues (ABET 2001).

(...continued at foot of next page)

Changes to SARTOR: Output standards

A fundamental review of SARTOR is currently in progress, with a view to remedy a number of shortcomings that have been experienced:

- IEng-accredited courses have not taken off as was hoped. The title and definition of IEng has caused much confusion and negative perception *(Wason 2002)*. One of the outcomes of the SARTOR review will be a revised definition and progression path for Incorporated engineers.
- The imposition of entry standards has been widely-criticised, and felt by some to have worsened the recruitment problem for engineering degrees - on the other hand, these standards seem to have given engineering a higher intellectual status in the perceptions of many school teachers and careers advisers. The new version of SARTOR will likely see a shift of emphasis from entry standards to output standards. This is being promoted by general trends in higher education in any case, particularly the QAA benchmarking project *(QAA 2000)*.

A focus on outputs rather than inputs is something that is generally welcomed in the engineering profession. This desire is reflected in the Engineering Professor's Council output standards initiative *(EPC 2000)*. A new focus is particularly welcomed by some engineering practitioners:

Why was the JBM requiring mathematics as an entry standard? The institutions should be demanding outputs and universities should do whatever they have to do to meet them. There are plenty of other weaknesses that graduates have, like spatial awareness, which the institutions have never highlighted. (Practising structural engineer)

What are the implications for mathematics in the development of output standards? As we noted above, there is already a shift taking place from teaching about engineering knowledge to teaching about engineering 'process', with design taking on a more central role. The move towards course accreditation based on output standards strengthens this change.

Should mathematical competence be an output of an engineering degree? The EPC document (only a provisional version at the moment) is interesting in its references to mathematics, because this is almost always paired with computing skills, for example:

…the ability to use mathematics and computing skills to create determinable models by deriving appropriate constitutive equations and specifying appropriate boundary conditions in the context of… (EPC 2000, p. 16).

One particular impact on mathematics might come through assessment processes, if not direct changes in curriculum content. It is common practice for mathematics (and other analytical, written-response) examinations to operate with a 40% pass mark, where it may be possible to 'scrape a pass' by picking and choosing amongst exam questions. Obviously, this is not consistent with courses designed according to output standards/benchmarks. The *IMA (1999)* report on mathematics in the SARTOR 1997 framework made some interesting suggestions in this respect, advocating for IEng-accredited degrees that core mathematics topics should be subject to 'high threshold criterion referenced assessment so that universities can guarantee that any incorporated engineer is numerate and algebraically literate' *(Croft & Lawson 1999)*. Interestingly, the IMA report did not advocate the same assessment practice for CEng degrees.

(International Comparison: USA, continued)

There may be lessons for the UK at the moment in the fact that US universities do take in students with GCSE-level mathematics and turn out world-class engineers as good as anywhere. However, it needs to be borne in mind that engineering schools represent a huge spectrum, and an engineering degree is expected to mean different things. Some schools do a level of mathematics beyond the UK undergraduate mathematics degree, whilst others will cover just the specified minimum. The system works by experience - employers effectively recruit from the schools that they know for the kind of graduate they need. The US system has nothing corresponding to the increasing levels of external accountability and standardisation being required in UK university teaching.

In the US, mathematics service teaching is a high status activity, which contrasts with the UK where service teaching is quite often the 'poor relation' to the teaching for undergraduate mathematics courses. University mathematics teaching, with Calculus as the core subject, in the US has been the subject of problems, with poor learning and engagement by students, as in the UK. What has been different there is the existence of a nationally-organised Calculus Reform initiative that has provided a focus for curriculum development, including the development of IT-based calculus courses. On the whole, mathematics education dialogues are more extensive and 'joined up' than in the UK: mathematics education research is definitely a part of the professional concerns of university mathematicians in the USA, helping to provide a theoretical perspective and principled basis to curriculum reform.

University restructuring and the politics of mathematics teaching

Student recruitment is not the only factor that has been bringing about changes at universities. Staff workloads have been increasing for many years, with the Research Assessment Exercise, and a general increasing emphasis on external research income, tending to skew priorities towards research. There have, indeed, been changes in the type of people appointed to engineering lectureships: we were told that it is now very difficult for engineers from industrial backgrounds to get lectureships, since there is so much emphasis on appointing lecturers with a research profile. This impacts in two ways: for all universities, it has become more difficult to find teachers of design with sufficient experience in industry; also, the 'post-92' universities are struggling to maintain their valuable tradition of recruiting part-time staff from industry to teach on the more vocationally-oriented (generally IEng-accredited) courses.

A trend occurring in many universities at the moment is reorganisation into schools, largely on the grounds of sharing resources and cost reduction. Along with the creation of schools of engineering, the question of who teaches mathematics comes onto the agenda, since a school of engineering can, economically, benefit from having its own dedicated mathematics teaching staff, rather than paying out to a mathematics department. (The implications for mathematics departments are severe - it is quite common for them to derive up to 50% of their income from service teaching.) Our perception is that there is little feeling amongst engineers to take over mathematics teaching entirely. However, communication between the servicing and serviced departments is not always ideal, and it is worth remarking that in a couple of universities we were told by mathematicians that they have tried to open dialogues with engineers but have had no success – it appears that engineers can be more conservative about mathematics than mathematicians! (There is also the problem of vice-chancellors imposing short-term changes to address financial problems and thus restricting longer-term dialogues about teaching.)

Conversely, the universities where we heard that experiences of engineering service mathematics teaching are positive are those where there was a detailed dialogue between departments, and we also noted that joint teaching is becoming more common. One interviewee, an applied mathematician, eloquently described the advantages of teaching by mathematicians:

The British university system has departments of 'applied mathematics', which were created to engage with the 'universal applicability of mathematics'. This gives us a range of expertise–we can use a certain technique to solve a problem in heat transfer, or in environmental problems–the same equation with different initial and boundary conditions. We can show that, where an engineer probably wouldn't be able to, only knowing the application in their own discipline. To have breadth of knowledge is part of the university experience; I think mathematicians can provide a better experience for students.

... Academics do not have a lot of time to work on their teaching. Mathematicians ought to be best placed to develop maths courses, compared with the new junior lecturer in the engineering department who's been delegated to teach 1st year maths. There are senior engineers who can do a very good job in teaching maths, but shouldn't such people be teaching engineering? ... I think engineering mathematics is important as a continuation with school experience, because engineering itself is a very new experience.

All of this is sound in principle, but it is apparent that mathematicians cannot be expected to invest effort to develop engineering mathematics courses in a highly-charged atmosphere. Even where the potential for dialogue is good, it seems that for this to work, there has to be a sufficient closeness of interests - as one engineer interviewee noted:

Mathematics is such a huge spectrum of understanding, with the 'logician' at one end, and us engineers at the other, using mathematics to model physical processes to learn things which we could not otherwise. You need someone to explain maths to you who is near to you on the spectrum. If you talk to someone at the wrong point on the spectrum you're lost. In my university we do have an engineering mathematics department which is keen to teach engineers, but they still think differently to civil engineers! (Lecturer in civil engineering)

International comparison: Europe

University education varies widely in style across the European countries. Also, mathematics-based degree subjects are subject to different cultural values about mathematics (for example, in France mathematics-based high-school programmes are the most prestigious and competitive). A useful point of contact for mathematical issues is the Mathematics Working Group (MWG) of SEFI (the European Society for Engineering Education, http://www.sefi.be), and we consulted several of the UK members of this group [the MWG has a website at http://learn.lboro.ac.uk/mwg].

Despite the widely-different educational systems, a recent report by the MWG *(SEFI 2002)* notes similar trends to those occurring in the UK across many European countries: a declining popularity for engineering degrees, at the same time as governments promoting increasing numbers of students going to university; and a decline in mathematical competence of students arriving at university.

The SEFI-MWG has sought to influence thinking about mathematics teaching by developing a '21st century' mathematics curriculum for European engineers *(SEFI 2002)*. This specifies a core curriculum for all engineering disciplines followed by more specialist options. A future publication of the MWG will be concerned with assessment. Practice varies widely across European countries: the UK's highly-complex written examination system contrasts strongly with the method used is some countries of the '10 minute interview', which is intended to gauge a student's competence if the right questions are asked. Given that assessment procedures do influence so much the styles of mathematics curricula, and the learning attitudes of students taking courses, the MWG intends to look at the different forms of assessment: what is aimed for by the different types, and if there a better way of achieving those aims.

2.3

The 'mathematics problem' in higher education

For some years now, mathematics has often been regarded as a huge problem looming over all mathematically-based degree courses. This has come to be known in the UK as the 'mathematics problem', originally characterised in a 1995 report, *Tackling the Mathematics Problem* *(LMS 1995)*:

There is unprecedented concern amongst mathematicians, scientists and engineers in higher education about the mathematical preparedness of new undergraduates… The serious problems perceived by those in higher education are:

(i) a serious lack of essential technical facility - the ability to undertake numerical and algebraic calculation with fluency and accuracy;

(ii) a marked decline in analytical powers when faced with simple problems requiring more than one step;

(iii) a changed perception of what mathematics is - in particular of the essential place within it of precision and proof.

Further reports *(eg Sutherland & Pozzi 1995, Barry & Sutherland 1998, Sutherland & Dewhurst 1998)* continued to gather evidence of a severe problem. The developing situation for engineering mathematics in particular in reflected in a series of conference proceedings *(Mustoe & Hibberd 1995; Hibberd & Mustoe 1997; Hibberd & Mustoe 2000)*. One report, *Measuring the Mathematics Problem* *(Engineering Council 2000)*, has had a particularly wide influence. This noted the very widespread use of diagnostic tests to measure the mathematical competence of new undergraduates, and the fact that gauging mathematical competence of students is now a problem in those (research-oriented) universities with homogeneous student cohorts (in terms of qualifications), as much as those universities that have always dealt with more heterogeneous cohorts. This report advocated the universal diagnostic testing of students upon arrival at university and that 'prompt and effective support should be available to students whose mathematical background is found wanting by the tests'. Several national projects have recently been funded to address these issues[5] by providing resources at a national level for testing and mathematics support.

It seems that not all of the observed problems should be directly attributed to changes in mathematics education. The declining popularity of engineering degrees, together with the government-backed push to expand student numbers, has led to the typical engineering class becoming very heterogeneous:

The ability and motivation range within engineering classes is large. This is mainly because of the unpopularity of engineering as a career. … It leads to low numbers applying to study engineering, resulting in departments accepting marginally-qualified candidates in order to keep their numbers up to survival level. Engineering classes therefore include extremely well-motivated and well-qualified students sitting alongside ill-qualified and poorly-motivated colleagues. *(Goodhew 2002)*

This tail of struggling students can be a large one - perhaps as much as a third of a first-year class, even where the students have satisfied the A-level points entry criterion (ie 18 points for BEng, so on average students will have grade C in A-level Mathematics).

[5] The UK Mathematics Learning Support Centre, and the HELM (Helping Engineers Learn Mathematics) project [http://ltsn.mathstore.ac.uk/FDTL/index.shtml]. Also the MathsTEAM project [http://www.mathstore.ac.uk/mathsteam].

It is agreed by all parties to the UK mathematics community that A-level maths no longer provides what it once did - a grounding for mathematics-based study in higher education. (We leave aside here what other roles A-level plays today, or should play in the future – this issue is being tackled at the moment by a DfES-commissioned Post-14 Mathematics Inquiry [6].) The mathematics problems of students arriving at university to study numerate degree courses fall into several areas. One is the 'patchiness' of A-level topics studied, due to the modularised curriculum–so that some students in a class will have studied, say, vectors whilst others have not. Only a relatively small core of material can be assumed for all students at university entrance (see the *Report Appendices* on the website for details of this core curriculum). It is quite common for students not to study any mechanics modules, and overall there is much less calculus knowledge and practice than was the case twenty years ago. In the Questionnaire that we distributed, calculus was identified as a near-universal problem area (see the *Report Appendices)*.

These problems are by no means new, but their extent has carried on growing. *Mustoe (2002)* points to a lack of response over the years on the part of universities:

There is a fundamental mismatch between what is expected of students and what they can realistically achieve; even some of the most able students find difficulty with concepts that would have been mastered by their counterparts ten years ago. Mathematics requires time in order to develop a body of useful techniques, and omission of important foundations causes problems later. Above all, there are no 'quick-fix' solutions. (Mustoe 2002)

Many engineering courses in the first term/semester of the first year will assume quite a sophisticated competence in calculus, say for structural theory of beams in civil engineering, or thermodynamics in chemical engineering. Whilst university engineering courses remain so 'knowledge-packed' (although as noted earlier there are moves away from this) any delay in commencing such core engineering courses has large knock-on effects to the course as a whole. However, to remediate student difficulties with mathematics in parallel with engineering courses is not at all easy, and demanding on resources and students *(Croft 2001; Mustoe 2000, Mustoe 2002)*. Below (section 3.3) we document some of the ways in which mathematics teaching is developing to deal with this problem.

Algebra and geometry

The introduction into schools of the National Curriculum and the associated GCSE examinations in 1988 saw radical changes in the nature of school mathematics. An immediate gap opened up between GCSE and A-level mathematics, and traditional approaches to algebra and geometry focusing on the 'technological tools' of mathematics (often baffling to a great many students) were replaced by approaches favouring the practical value of mathematics in everyday life. These changes and their consequences for mathematics learning are detailed in two reports by the *Royal Society (1997, 2001)*. *Sutherland (1999*) makes the case for the necessity of an engineer to possess 'symbol sense', and provides an overview of how the school mathematics curriculum has drifted away from providing it:

Much of classical school algebra involved teaching standard algorithms for solving equations. These standard algorithms have become associated with traditional and rote ways of teaching which have been perceived as being in opposition to understanding. Understanding has become the almost unquestioned aim in mathematics education. The unintended effects of this celebrating of 'understanding' has been that the majority of pre-16 pupils have had very little experience of symbolic algebra … There is a strong argument for saying that engineering education should enable engineers to develop a sophisticated symbol sense within the context of solving engineering problems. It is also clear that it is extremely difficult for them to do this if they enter university without a background confidence of working with symbols. Sutherland (1999)

Algebraic fluency

Algebraic fluency, if missing, is very difficult to develop in a university setting. *Barry & Sutherland (1998)* tested students' algebraic competence across all the years of the engineering programmes at the University of Bristol. One particularly striking result was that performance, comparing first year with third and fourth year students, did not improve over time. This persistence problem was corroborated in a number of our university interviews, for example:

I know a lot of students who have problems with algebra, and although they will succeed and get a degree, you can't honestly say that they have gained fluency. I recall one student who worked very hard, mastered what was necessary, avoided more mathematical optional courses, and got a first class degree - but I could tell that the fluency was not there in basic things. Or another student - who was doing employer numeracy tests, and came to me for help with doing percentages - just before passing a degree in electrical engineering! (Mathematics lecturer, specialising in teaching to engineers).

What about the perceptions of employers about algebra? Our data on this is limited, and we do not wish to generalise too much from it. Yet in our previous research study *(Kent & Noss 2002a, b)* our observation was that most civil engineers that we observed (excluding the small number of specialists in analytical methods) did very little algebraic manipulation. Algebraic formulae do appear in the context of building codes and other standardised knowledge, but their use is mainly by substituting values into parameters (or, putting the formulae into a spreadsheet and calculating various results for different parameter choices). It could be argued that the engineers' lack of algebra-use is due to the problems of mathematical preparation that we have just described; however, the message we received from practitioners was that what matters is not manipulative skill by itself but an 'holistic awareness' and ability to recognise where mathematical work is required:

There are really only two groups of engineers who can do serious hand calculations: those within two or three years of graduation, and the lifelong analytical specialists. What most engineers retain in the long term is not the ability to execute maths, but knowing that methods exist, and who or what you can go to find a solution. (Senior structural engineer)

The crucial issue is in the easy and comfortable coupling of the real world and its definition in mathematical formulae. An essential feature of any professional is to know when he or she needs help (e.g. solicitor seeking counsel, GP referring a patient to a consultant), so it is with engineering, the skill is in knowing the point of referral to a person of greater analytical skill than oneself. (Lecturer in civil engineering)

Geometry and visualisation

Of particular concern for civil and structural engineering is the topic of geometry, both for practice and academia. This is an area of the 'mathematics problem' that has received less attention than algebra. We perceive that the term 'geometry' means different things to different engineers, and can distinguish three main aspects to this:

- broadly-based spatial awareness (2-D and 3-D visualisation)
- properties of triangles and trigonometric formulae
- education in the systematic structure of mathematics

The first aspect was commonly mentioned by practising engineers as a problem area, particularly in its relation to the visualisation of three-dimensional structures and two-dimensional plans:

We have had graduates who generally were weak in reading drawings. And I think one reason for that was that their geometry was weak, to calculate and visualise in three dimensions. Say, you took a section through an unusual plane in a structure. (Design department of a large building contractor)

The second aspect was also mentioned by one practitioner ...

The problem with trigonometry is being able to calculate areas, volumes, say in temporary works for a construction, to calculate the size and position of an access tower. We have trained people up quite successfully, but I feel it's unfortunate and unnecessary, I would like to see people come through with some trig, some geometry, and then mechanics, both statics and dynamics… (Design department of a large building contractor)

... and it was commonly referred to by engineering academics (cf also responses to the questionnaire):

The chief deficiencies at entry, in trigonometry, are associated with the ability to sketch graphs of functions and the fact that they have never had to commit any trig formulae to memory. The process of memorising the formulae usually forces the brain to see the patterns and linkages. (Lecturer in civil engineering)

It is striking that, for mathematicians, 'basic' trigonometry is regarded as a mundane topic on the way to the 'interesting stuff' in calculus, whereas for engineers it is an absolutely crucial component of, for example, thinking about structures.

The third aspect is well described by *Blockley & Woodman (2002)*:

Many of us over 50 enjoyed Euclidean geometry and the beauty of theorem proving. This is no longer in the syllabus. However all mathematics is the ultimate form of logical rigour. This is certainly a quality required of engineers…. In these modern times, when people are increasingly relying on 'bullet point' presentations, the ability to work though a set of ideas using a strongly logical mind is of very great importance. (Blockley & Woodman 2002)

Geometry is not only a 'mathematical way of thinking', but also an organising principle for mathematics. *Roper (1997)* argues that, before the curriculum reforms of the 1970s ('new maths') and 1980s (practical maths, numeracy), geometry was not only the vehicle for proof (logical thinking) but also the organising theme for the 11-18 curriculum:

Euclidean geometry, for all its faults, provided the roots and trunk of the 11-18 mathematics syllabus. The calculus was anchored to it, and the two together provided a coherent body of mathematics in which proof and the requisite algebraic manipulation could develop. … My contention is therefore that the major reason for the lack of algebraic manipulation skills is that there is actually no call for them at A-level beyond the level of simple technique. … Stressing the lack of manipulative skills, introducing papers where calculators are banned, will not help unless there is a coherent body of mathematics in which these skills are essential and can be used with intent. Calculation and manipulation are pointless activities in themselves' (Roper 1997, p. 20)

Roper's point has strong implications for schools, of course. But might there be some implications for university engineering mathematics too? In the case of civil/structural engineering, could geometry be made more of an organising principle for engineering mathematics courses, which could be linked into the role of geometry as an aspect of, for example, 'structural feel' in structural engineering? It's ironic that mathematicians see geometric structure in all of the areas of mathematics studied by engineers (calculus, differential equations, linear algebra, etc), but this is hardly taught to engineers (and in advanced courses, if at all), whose primary encounters with mathematics are strongly symbol-based. This certainly contrasts with the visual-geometrical nature of engineering design, which emphasise the need for students to sketch, to visualise and to engage with physical models.

Confidence in mathematics

I feel people come from school frightened about maths. (Civil engineering lecturer)

The 'mathematics problem' is usually described as a skills problem, but this has two aspects: the knowledge of mathematical techniques/facts, and the confidence to make use of them in an engineering context. We think this distinction is important because it influences how one might go about addressing the problem: to develop students' mathematical confidence is a slow process, which cannot be achieved through quick remediation, unlike the problem of 'filling in' some gaps in mathematical knowledge. It is known that many students experience A-level mathematics topics as modularised and disconnected pieces of knowledge, and this experience can be compounded in cases where the initial university mathematics course is also lacking in connections - either to school mathematics or to the students' engineering discipline.

The gap between school and university mathematics is both in curriculum content, as described already, and in 'culture'. Indeed, it has been questioned whether academics are the most appropriate teachers for first year mathematics. *Sutherland & Dewhurst (1998)* suggest that, as universities are required to teach more and more of the mathematics which in the past was taught in schools, 'the teaching needs may be more akin to school teaching and university lecturers are not necessarily the best people to be carrying out this kind of work' (*Sutherland & Dewhurst (1998), p. 21*). Also, a case was made to us that mathematicians tend to make a better contribution than engineers in matters of confidence:

There are engineers teaching maths in engineering departments, but they are always good mathematicians by background, and I don't think they understand today's students, who will struggle to be anywhere near their level. We do have to get the foundations laid - not assume that because someone *got an A-level grade C they are proficient in mathematics. That needs someone to teach it who really understands the students. (Mathematics lecturer specialising in engineering mathematics)*

Connections into engineering are also vital, as one mathematician reported to us:

There's been a problem of regarding mathematics as a distinct thing, like structures or fluids, when it needs to be part of the engineering spectrum. So lecturers in structures complain 'if I try to do some maths in my lectures, I get told off because that's the job of the maths lecturer'. Getting more integration is to be welcomed … Students consistently want to know, 'where will I use this mathematics?', and it helps if you can say, this will matter in structures, for the buckling of a strut - this equation will arise, and this is how you solve it. So, I've had to learn civil engineering, to know where all the stuff comes in, and also to know the real relevance - if I just read a book, the engineers might tell me 'we don't do it that way any more' or 'we use a different sign convention here'. (Mathematics lecturer)

Confidence in mathematics needs to develop early, of course. The weaknesses of school preparation have been recognised by the mathematics community and steps are now being taken to address them; notably, GCE A/AS Mathematics is currently being reviewed by the QCA, and the whole of post-14 mathematics is the subject of a review commissioned by the DfES. Any changes in school mathematics will take a considerable number of years to have an effect, and meanwhile there is a danger for mathematicians and engineers in universities to argue themselves into a corner about what to do about it, to seek to return school mathematics, and therefore beginning university mathematics, to 'how it used to be'. Even if this were possible, would that be the best mathematical preparation for a 21st century engineer?

One reason for arguing 'no' is that in the work of practising engineers there are now many 'symbolic fluencies' in use- that is, the expression of mathematical relationships in the various symbolic forms provided by computer software, alongside traditional pen-and-paper algebraic symbolism- for example, general-purpose tools such as spreadsheets, numerical packages and computer algebra systems as well as specialist engineering software (structural analysis, geotechnical analysis, finite elements, etc).

The possibilities of these 'new symbolisms' need to be critically examined in engineering education, especially considering the diversity of engineering courses that the engineering community is seeking to develop, and the different levels of mathematical sophistication that will be involved:

- To what extent should pen-and-paper fluency be the primary basis for developing an understanding of engineering principles, given the availability of 'alternative symbolisms' in mathematical software (spreadsheets, computer algebra systems, etc)?
- Is mathematical fluency a prerequisite for introducing engineering principles, or is it the case that *some* mathematics can be postponed until later?
- Are some ideas best introduced by mathematical symbolism?
- How can working with alternative symbolisms help students to gain a better understanding of conventional mathematical symbolism?

3

Change and innovation in engineering mathematics: Issues and examples

3.1

Engineering degrees and the place of mathematics

Current degree programmes fall into three types, MEng, BEng and BSc; the first two of these are CEng-accredited and the third is IEng-accredited. It is by no means certain whether, in the restructuring of degree courses and professional status that is under way, these will remain the only possible 'paths' into the profession. There has also been some debate about a more radical change in terms of revising the definition of CEng to a cover a rather broader range of professional capacities than currently, and this might have implications for the analytical elements of CEng degrees, and the necessary mathematical training required.

We would like to consider first the place of mathematics in the degree programmes as they currently operate.

Mathematics in 'less technical' degree programmes

There is a widely-agreed need for broader, 'less technical' courses for civil engineering, which in the form of BSc degrees (with IEng accreditation) have not taken off at all well, yet the industry is now experiencing major shortages of this kind of graduate. For such students, there is a generally acknowledged need to teach mathematics in less depth but still to convey meaning and understanding. There is significant scope to reassess mathematics teaching in this context: an interesting proposal made in a report by the Institute of Mathematics and its Applications *(IMA (1999)* (see also *Croft & Lawson (1999)*), advocated a much more explicitly holistic approach which seeks to establish a mastery of mathematical techniques taught through engineering contexts, and with the integrated use of mathematical software (computer algebra, graphical calculators, spreadsheets). BSc students tend to show a great diversity of mathematical backgrounds, with GCSE at a minimum and a range of post-16 qualifications. Students can be expected to have serious weaknesses in their mathematical backgrounds, and expectations of what can be achieved should be realistic: 'a broad range of well-understood lower-level mathematics is more valuable than poorly grasped higher level material' *(Croft & Lawson 1999)*.

As part of our consultation exercise, we looked at two particular BSc degrees in civil engineering. Both start from a base of GCSE Mathematics, although in one case it was reported that even basic numeracy could be weak and needed to be revised. Both degrees have mathematics taught by engineers in the first year, then mathematicians in the second year, to ensure that the relevance to engineering is dealt with early. In one course, there is a 'high threshold' test early on: 'we start the maths course with basic arithmetic, and then there's an assessment where the students must get 100% - taking it as many times as they need; that assures us that they've got the basic tools to move on.' Our impression is that the mathematical aspects of BSc degrees are functioning quite well, by building from a base of GCSE, and setting a lighter loading of analytical courses. The principle challenge with such IEng-accredited degrees seems to lie in actually attracting more students to study them, and persuading employers to recognise better the concept of IEng (or, whatever classification might end up replacing it in the forthcoming revision of professional career paths).

Mathematics in BEng and MEng degree programmes

The debate around the development of BEng and MEng degree programmes is much more contentious than for BSc degrees. Whereas there is a broadly -accepted need for BEng/MEng degrees to change in terms of introducing more design-oriented studies, we found considerable resistance to the idea of changing mathematical content, or mathematical entry requirements. A common response was 'we need to teach core subjects - structures, fluids etc - from the start of first year, so we need students with a knowledge of calculus from the start of first year'. (Of course, this contrasts strongly with the 'less technical' aspiration of BSc degrees.)

However, this does not mean that courses are not changing in terms of mathematical expectations. Given the common problems of lack of mathematical fluency that students experience, there is definitely a tension between the need to maintain 'rigour' and the growing difficulties of students to handle this via mathematical symbolism. It was suggested to us that current students are becoming 'less receptive', and 'less able', to extract the key principles and ideas from heavily symbolic/analytical courses. Perhaps the most-discussed area for this debate is structural analysis, where, over the last 20 or so years, syllabuses have been gradually modified to have reduced analytical content, and more emphasis on the use of computer software- leaving a sense of disquiet amongst academics about where things are going *(May et al 2002, Blockley & Woodman 2002)*. Engineering is not alone in this kind of change, of course - it is happening across all science, engineering and technology subjects. It is also the case that mathematics courses for engineers and scientists are changing, with much material that used to be considered a prerequisite from A-level now being taught. Indeed, given the number of universities that find it necessary to re-teach basic calculus, we wonder if it is reasonable in the long term to go on expecting 'calculus from day one' in engineering courses.

The engineering institutions' response to these difficulties with mathematics, via the JBM, has been to remove the A-level mathematics requirement (temporarily) and invite civil engineering departments to re-consider their options. Of the civil engineering departments we surveyed, all except one said that the change could not alter their admissions policy for BEng/MEng, because the mathematical requirements of the courses demand A-level knowledge as a prerequisite. For the exception:

We could take someone in with AS maths, and bring them up by teaching more maths in the first year - that is an option. We could take in a lot more students that way, but being very selective - if the overall A-level profile is right, satisfying the SARTOR requirements -I do think it is harder to get a good A-level profile if you take maths and physics compared with other subjects.

The circumstance of the department in question is that it runs a full range of courses (BEng, BSc, Foundation degree), it is used to working with students from diverse educational backgrounds (it is a 'post 92' university), and so it has the experience and infrastructure to consider this change; flexibility is important - so that a student may enter on one degree course and gain promotion later if they perform well or be invited to demote themselves if they do not perform well. A difficulty for universities which only teach BEng/MEng is that not only do they lack experience and resources, but there is often considerable resistance to the university being seen to offer any kind of 'less academic' degree course.

What kind of mathematics in engineering degrees?

We conclude this section by making a rather more contentious point. The picture presented to us in our discussions with many engineering academics was that there are two distinct kinds of mathematics for two kinds of engineering degree - that BEng/MEng students require a 'first principles' approach to analysis, to deal with non-routine problems and tasks, which would not be required of BSc students after graduation.

We would like to question this. Our perception of practice is that there is a *spectrum* of mathematical competence, which is based on the *same* characterisation of mathematical understanding, such as *Nethercot & Lloyd-Smith's (2001)* description of mathematics as 'a medium for communicating concepts, ideas, and information in a parallel way to text- and not as the mastery of a series of abstract processes with little or no linkage to physical understanding and application'.

What might the 'spectrum' mean for engineering education? It is arguable that the spectrum is something which develops later, based on the common preparation of an engineering degree–as graduates develop into practising engineers, gravitating towards or away from detailed analytical work, and developing (or forgetting!) the detailed mathematical competence required.

We think this mathematical question is in fact embedded into the more complex debate about the kinds of engineers that an undergraduate education seeks to develop. It seems that the present model aims to produce a 'balanced' graduate who then elects to develop in different ways. Yet, as we described earlier, there is much discussion in the profession about how well this model works and whether a greater degree of diversification should be allowed within degree programmes.

3.2

IT and engineering mathematics

Doing structural analysis by pen-and-paper time after time gave you an understanding, but the same thing can be done on a spreadsheet. You can tune the input numbers and watch the result. Even if you don't know what's going on, so long as you can rely on the computer's calculations then you are developing an understanding. I don't think many academics have learnt themselves that way, yet. For the first time in history you can try something and see what it does without damaging anything. It's the classic learning tool, video games are based on that principle: you crash a car, have another go and learn not to crash it. So with structures, you play around with a computer model of a bridge, overstress it and watch it collapse, underbrace it and watch it vibrate. You never before had the time or the money to do that. (Practising structural engineer)

As we noted in Section 1.3, IT has revolutionised the use of analytical techniques in civil engineering practice. More recently, it has begun to make deep impacts on analytical courses at undergraduate level–as in the quote above–though this is still a controversial change. This can be seen, for example, in the debate on the teaching of structural analysis that has been running through structural engineering journals and conferences for the last decade (see, for example, *Allen 2000a)*. For example:

Under the onslaught of [computational] power questions arise; 'what should be taught in structural analysis courses'? 'Is it now possible for the outcome of a structural analysis course to be achieved just by teaching students to run a computer package'? Although we do not believe this latter, rather extreme, assertion to be true nevertheless we do think it is timely to ask what should be the objectives of a theory of structures course and, consequently, what should be included in such a course. … as the old outdated analysis techniques fall through the sieve of relevance we have to determine what fundamentals need to be retained and re-incorporated into a contemporary syllabus. (May et al 2002)

In the area of mathematics courses, revolutionary possibilities have been discussed for some time, and software in the form of general-purpose computer algebra (symbolic manipulation) systems (eg Mathematica, Maple) is now available to challenge the mathematics curriculum in ways similar to those that obtained in structural analysis. However, engineering mathematics courses have not yet gone through widescale changes. Perhaps it is just too soon for the software to have had its effect. University mathematics departments tend to be under-resourced, so in the case of service mathematics it is a very difficult proposition to take hundreds of students out of lecture rooms and into computer laboratories. The use of CAS in schools remains very sparse (since, unfortunately, A-level curricula and examinations have not yet developed a coherent position towards use of IT), so that there is very little ground on which to begin teaching at university:

I've seen a lot of research on teaching with software, but I see problems with implementation. You have to have the culture of using software so that students are switched on to it, and provide an infrastructure of training and support. We use software very successfully in other subjects, for example soil mechanics. We don't do much with maths at the moment, but we're happy to learn from good practice elsewhere. We do already have Matlab on the network for student use. … My personal opinion is that for successful use of mathematical software in university, the culture needs to develop in school. We teach maths from the start of first year, whereas we introduced other software, for structures, soil mechanics, in the second and third years when students are more mature. (Lecturer in civil engineering)

Of course, there are significant dangers in losing the teaching of pen-and-paper mathematical techniques to 'button pressing'. As in structural analysis, a 'sieve of relevance' needs to be applied *(cf. SEFI 2002)*. In the accompanying box (Rainbow Bridge) we give an illustration from our own research of how mathematics and IT might be combined, using an example context from structural analysis.

The Rainbow Bridge *(Kent & Noss 2000)*

This example concerns collaboration with a civil engineer on the development of some simple activities for first-year students using the computer algebra system, Mathematica. The idea was to help students begin to get a 'structural feel' for how structures behave by letting Mathematica take the mathematical strain in some structural situations. (Please note that this is not intended in any way to replace the students' learning of semi-qualitative methods such as force diagrams for structural analysis–rather the intention is to motivate the use of mathematical software, and mathematics, for engineering analysis in a 'fun', informal example.)

In the 'Rainbow Bridge' activity, two pre-written Mathematica functions generate animations of a test load moving across two different simple bridge structures; at each step in the animation, the colour of each of the struts in a bridge represents the magnitude of the force in that strut induced by the test load (Figure 2).

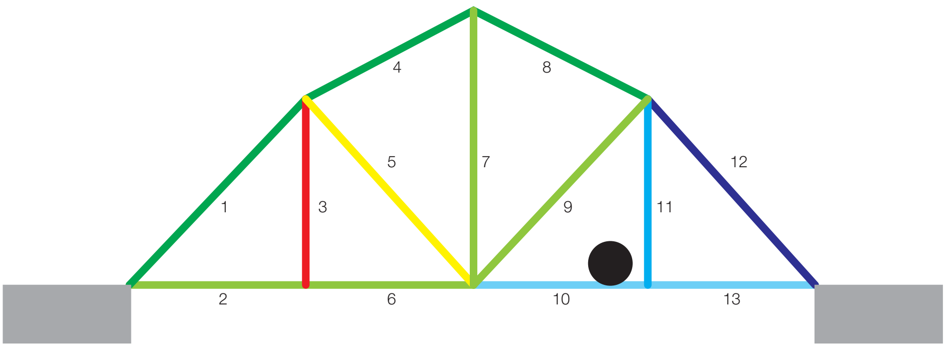


Figure 2: Frame from a 'bridge movie'; a test load (black disc) moves across a (2-D) bridge, whose struts change colour according to the magnitude of the force caused by the test load.

In the interests of having the students develop 'structural feel', they are not required to understand the Mathematica details of how the animation is generated, nor the mathematical details of how the strut forces are calculated (which involves the solution of systems of linear equations - something they have not yet studied). What they do have control over is the magnitude of the test load and the 'colour function' which maps a numerical force value onto a range of output colours. They are asked to consider how to design a colour function which yields the most useful information about what is going on in the bridge as the test load moves across it, and to design a function which would allow them to detect the maximum safe load that can cross a bridge given a maximum safe force for any strut.

It should be clear what are the engineering lessons from this activity: the students can get experience in how the patterns of forces vary in a loaded structure, and they are invited to consider, albeit for a toy example, a central engineering design question of determining what loads a given structure can safely support. The mathematical lessons may not be so obvious. Indeed, one might ask, where is the mathematics at all? Isn't all the relevant mathematics hidden inside the Mathematica functions? In the most obvious sense, the mathematics of the problem is indeed hidden–the solving of systems of linear equations. But in fact, mathematics is made visible in the interplay between the need to define colour functions and the high-level visualisation. For example, the 'maximum safe load' question demands the use of some form of piecewise-defined function.

This process of making and criticising representations (ie the colour function mappings) is not conventionally recognised as mathematics, at least not for non-expert, beginning students. But, we suggest it is entirely consistent with a modelling-oriented approach to mathematics - inviting students to think about the 'interface' between mathematics and engineering (see *Kent & Noss 2002a* for a development of this idea). A characteristic of the modelling approach is that early exercises, such as this one, need to provide a lot of support for the students, so that mathematical details do not dominate - as the students learn more mathematics, the amount of support decreases.

The general lesson that we take from examples like Rainbow Bridge is that it is essential to use a general-purpose mathematical tool (such as Mathematica) for this kind of activity. A structural analysis package might be able to offer the same output functionality, but crucially it could not offer the students the opportunity to inspect (later) the mathematical details - it would be the 'mathematical black box' which is so often criticized as a dangerous use of IT in engineering education. It would also be possible to do the task with spreadsheet software - but this lacks the capability of general symbolic manipulation, and it is crucial for students to engage with symbols - not least because so many students are reported to have difficulties with symbolic manipulation. We are convinced (though we are not aware of much systematic evidence) that in using a symbolic tool like Mathematica, students can improve their general understanding of algebraic symbolism, and develop a greater confidence with pen-and-paper (and mental) symbol manipulation.

3.3

Some examples of revised approaches to mathematics teaching

We present here four brief examples to illustrate the range of responses being made to problems with mathematics. Some additional evidence on what departments are currently doing is available in the responses to the questionnaire survey of UK civil engineering departments which we conducted (*see the Report Appendices on the project website)*. Generally, we found much disquiet amongst engineering academics about the state of mathematics teaching, but curriculum changes in the main are taking the form of gradual 'curriculum shift' rather than widescale systematic change. Whilst not all engineers agree that widescale change is required, we do expect to see much more activity going on within the next few years, in particular as the developments with engineering regulation (SARTOR) and revised A-level mathematics curricula (Curriculum 2000 - including the split into A and AS level courses) impact on universities.

EXAMPLE 1: Curriculum shift and remediation

As the 'mathematics problem' has developed over the 1990s, mathematics courses have, in the main, been gradually adjusted to deal with students' difficulties. Here are some typical comments on gradual change:

For our BEng course, we assume a knowledge of A-level maths, but the assumed level has changed dramatically - the first year now involves maths which we would simply have assumed 10 years ago. And that has a condensing effect on the amount of maths students can do during the course. (Lecturer in civil engineering)

Students now are doing about 3/4 of the maths that was in our courses 10 to 20 years ago. And we have to start at a lower level; we can assume very little experience with differentiation and integration. (Lecturer in engineering mathematics)

Alongside such curriculum changes, various measures of remediation have also developed. 'Mathematics support' centres are now operated in about 50 universities - offering mathematics learning resources and personal, informal mathematics tuition. These efforts are now developing on a national basis, so that ideas and resources can be readily shared. The most popular resources tend to be paper-based; although computer-based packages have been produced over the years, exploiting the possibilities of multimedia, these tend to age quickly in appearance (also in compatibility with current technology) and cannot easily measure up to students' expectations, often based on state-of-the-art computer games. One new computer-based product that is currently being evaluated in a number of universities is the 'Maths for Engineers webdisk' [7] ; this has taken a number of existing, tried-out mathematics learning resources (videos on modelling topics and mathematics tutorials, and revision/exercise courseware) and put them together into a professionally-integrated package on a DVD-ROM.

There is a growing body of opinion that attempts at remediation can only be a sticking plaster to treat the symptom, not tackle the fundamental problem. For example, some mathematicians have advocated a more integrated approach to mathematics in engineering:

It is suggested that teams comprising both engineering and mathematics staff draw up mathematics and other engineering syllabuses which allow the mathematics modules to get a head start before the other engineering modules make use of that mathematics. This will allow the students to gain that all-important ingredient, confidence, in mathematics while becoming enthused about the other engineering subjects. Then these subjects can be revisited with the necessary mathematics in place to allow a more analytical study. (Mustoe 2001).

This proposal seems to make a good balance with the 'design-based' approaches that are increasingly shaping engineering degrees (see below).

EXAMPLE 2: A 'multi-level' service mathematics course for engineers

There are instances of successful curriculum innovation where there has been little recourse to radical changes. For example, the Mathematics Department at UMIST has dealt with the problem of wide mathematical ability in new students by developing a three-level, streamed teaching system for all the service mathematics courses run at UMIST *(Steele 2000)*. Students are assigned to fast/medium/slow mathematics courses on the basis of an initial diagnostic test (students on the slow stream lose the opportunity to take an optional module later on in the course).

[7] http://www.mathsforengineers.net/

The feedback from students is that they are happy to be assigned to work in different mathematics courses, and certainly the feeling of the civil engineering academics is that the mathematics course works well for their students, with a significant levelling up of mathematical ability by the end of the first year. They also like it that mathematics remains to be taught by mathematicians so that the students can experience a 'different way of thinking'.

EXAMPLE 3: 'Engineering first and mathematics second'

We present two examples of an approach that responds to a question we raised earlier: Are there less mathematically-oriented (and more engineering-oriented) ways of teaching that can be used as an initial introduction to engineering principles, to be followed later by more mathematical treatments?

Otung (2001, 2002) describes an approach called 'Putting engineering first and mathematics second' in the teaching of communications engineering. The idea is:

...to introduce each engineering topic using such means as lucid graphs and diagrams, intuitive reasoning, analogies, computer simulations, etc. to give the student an unclouded insight into the engineering concept and the underlying physical considerations, a clear appreciation of the parameters involved, and a good feel for the interplay of these parameters. This type of first encounter with the subject would not only stimulate the student's interest but it would crucially erect knowledge pegs on which they can hang a more precise subsequent discourse involving the language of mathematics. (Otung 2002)

Otung contrasts a 'maths first' approach, which adopts the language and symbolism of sets of orthogonal functions, with an 'engineering first' approach, that uses far more verbal and graphical argument, using graphs of sinusoidal signals to illustrate the effects of their (orthogonal) combination.

Dr Lawrence Coates (Civil Engineering, Birmingham University) provided us with an example of a similar approach in first-year hydrostatics. He contrasts an engineering-first approach, which takes the simplest case of a situation (constant fluid density), and this removes the need to use calculus. The mathematics-first approach is calculus-based, and more general, but risks obscuring the engineering principles by algebraic symbolism.

Both these examples struck us as surprising in the degree of formality of the maths-first arguments. While we, as mathematicians, might want to defend the power and possibilities of mathematical language for precision and compactness, we recognise that this cannot - and need not - be achieved at the expense of conveying engineering principles. Indeed, we were surprised to be told that the engineering-first approach was vigorously opposed by engineering educators.

EXAMPLE 4: Mathematics and problem-based learning

An interesting approach to mathematics is developing at the Manchester School of Engineering (University of Manchester) where the degree programmes for mechanical, aeronautical and civil [8] engineering in academic year 2001-02 have begun to adopt a problem-based learning (PBL) approach *(Lennox et al 2002)*.

The first two years of the course are PBL-based (reverting to lecture-based courses in Years 3 and 4): students are grouped into teams of eight, and work on a series of substantial engineering 'problems' through the year. Each problem is studied for one to two weeks and requires researching various aspects of the problem (including engineering principles) and negotiating a group solution. Some basic topics are still taught in lecture format: mechanics, thermofluids and mathematics. The approach to mathematics has kept the content traditional but the teaching format is 'PBL style', that is, students work in their regular teams, and submit the continuous assessment as a team (the main assessment is by individual tests and end-of-year exam). Part of the point is to keep the maths course in a familiar style for students newly-arrived from A-level study, and the course is still taught by staff from the Mathematics Department.

Compared with the previous course, the mathematics curriculum has been halved - this was based on asking lecturers 'what maths do you really need to teach your course?'. It is now very lean - basically calculus, some differential equations, but no matrices, complex numbers, Fourier or Laplace transforms, etc.

[8] Actually there are not many civil engineering students on this course, and these transfer to the Manchester School of Construction – a joint school between the University and UMIST – in the second year, which is where the majority of civil students enroll from the beginning of their studies. This is a transitional aspect of a full-scale merger between Manchester University and UMIST that is expected to take place within the next few years.

We found that it is maths which really puts students off in the first year, and a lot of the things in the syllabus seemed to be there for historical reasons. For example, we used to teach Laplace transforms for four weeks of the first year maths course. The only time they are used is once in a second year course, where we'd assume the students didn't know anything about Laplace transforms and introduce them in half an hour. The initial four hours of theory was superfluous, because it's the application we're interested in. The maths course used to show them how to solve with the transforms, but we have tables available, and we would never expect a solution without using tables, or using computer software.

In the first year there are 10 mathematics sessions of four hours each, which consist of a short lecture, a session for skills practice, and then a group-based problem. In the second year there is no mathematics course at all - it is addressed in specific engineering courses where the need arises. What underpins that is an expectation on the student that 'what I tell you is not all you'll need to know' - there are aspects of mathematics that they have to find out for themselves - but that is built into the culture of PBL from the start (eg it has been noted that use of the central library by first-year engineering students has gone from virtually zero to heavy):

One PBL exercise had quite a high mathematical content, involving a 1st order differential equation, which the maths course had not covered at that stage. So they had to go and find out how to do it - we did find the problem that each group of eight students had two or three who were good at maths, and the others in the group tended to rely on these to solve the mathematical problems. We didn't get as much collaboration as we would have liked in terms of the others picking up the maths - some were happy just to have the problem solved for them. The way the marking was designed last year meant that there were no penalties for doing that, which this year we have corrected.

Feedback on the course from staff and students has been extremely good. Remarkably, mathematics has gone from being one of the least liked subjects to being one of the most liked by students. In the past, a common student attitude developed that 'engineering is about applying maths, and it's very dull'. That created dropouts and was very depressing for the staff - things would only start to improve when students got into the third year. The students now are much more motivated; they worry that they're not learning enough, but the staff can see from observing the groups that they are learning well, and in some cases going beyond the level expected. The mathematicians teaching the course have been impressed by how well it works and intend to adopt the same teaching style for their Foundation mathematics course.

Other examples of engineering PBL are being developed with the support of the IEE, which has been taking an interest in PBL for some time. An initial working party report in 1998 has led to the development of a consortium of universities that are implementing PBL-based degree programmes, supported by a grant from HEFCE and industrial sponsors:

It is proposed that from 2004, MEng and BEng Electrical and Electronic Engineering degrees courses at UCL, UMIST and the University of Bristol will incorporate 'Problem-Based Learning' (PBL), a student-centred mode of learning which prepares students for life-long learning. The initiative … comes in response to concerns voiced by employers that many electronics graduates lack essential skills. PBL develops these skills without lengthening degree programmes. (IEE press release, January 2002)

The intention at UCL, for example, is to introduce a full PBL programme in Years 3 and 4 of their MEng degree. Not all course modules will change, the design-based ones will move to PBL, but theoretical ones will remain largely lecture-based. There is a recognition that the first two years of the degree will need to make moves towards PBL-style working, including the teaching of mathematics - the details on this are still emerging. Despite all the uncertainties, there is certainly a feeling at UCL that this is the way for engineering education to go, that their highly-qualified students (28+ A-level points) may possess a technical ability to do mathematics, but experience many problems with integrating mathematics into modelling and problem-solving; many students get used to the 'listen and regurgitate approach' which traditional written examinations encourage *(Kenyon et al 2002)*; *Spencer-Chapman (2000)* has reviewed the case for PBL in civil engineering, with similar conclusions. They are also concerned that too few of their students perceive themselves as 'becoming engineers' instead of passing exams to get a degree. This is also a problem for civil engineering: the proportion of MEng/BEng civil graduates in the UK who do not pursue careers in civil engineering is depressingly high.

3.4

Mechanisms for change

There are considerable opportunities for change. There are, of course, problems and difficulties brought about by changes in the mathematical preparation of students at school, in degree course priorities, and in changing professional practices: reacting to such changes is complex and often painful. But there is also a readiness in departments to embrace change proactively, to reconsider existing practices and to exploit new possibilities offered by new pedagogies, and by rebalancing the relationship between mathematical and engineering knowledge. In this section, we consider some of the mechanisms for change that are presenting themselves.

Like most subjects, engineering mathematics curricula often contain topics that are present for historical reasons, which are no longer used in engineering courses as in the past. These glitches can easily arise where there is a lack of dialogue about mathematics teaching–relationships between engineering and mathematics departments take a number of forms, ranging from mutually supportive to hostile! Perhaps, therefore, common ground can be gained by a constructive dialogue on two fronts: on the mathematical topics in the curriculum, and at the same time, on delivery and pedagogical approaches. It is our belief, on the basis of our modest survey, that these two issues are intertwined, and that consideration of one without the other leads inevitably to misunderstanding and inertia.

Pedagogy *is* highly constrained by resources available - it is relatively cheap for one person to lecture to a group of 100 students, and relatively expensive to put those 100 students into computer labs with adequate tutor support. However, given that engineering courses are going down a trend of being increasingly ‘design-based’ (employing design as an organising principle), with a decreasing use of ‘chalk and talk’ pedagogy, mathematics courses will have to accommodate themselves in some way to this trend. As the example at Manchester University shows (Example 4 above), this does not have to be a totally radical departure from existing mathematics teaching.

A number of models have been proposed for design-based learning. Problem-based learning is a ‘radical’ form of this, and the methodology has been criticised for civil engineering, because of its origins in medical education: the argument goes that PBL is good for ‘fact-based’ knowledge, such as medicine, but is less appropriate for an analytically-based subject like engineering. An alternative model, studio-based learning, is being developed by number of universities *(eg Eyre 2000, Irwin 2000)*, where teaching methods are moved closer to those of architecture schools. Whatever the particular methodology chosen for design-based learning, the effect on mathematics is similar: in the context of design, the need for analysis is ‘pull, not push’ - the need for analysis can emerge where design requires it, it is not pushed onto (or into) the student prior to having a meaningful context for it. (We are not advocating here the removal of mathematics teaching as a distinct subject in itself - there needs to be a balance between mathematics taught as a ‘pure’ subject by itself and mathematics ‘pulled into context’.)

In fact, there are a number of advocates amongst mathematicians for this kind of engineering teaching *(eg Mustoe 2001, IMA 1999)*. In the past, a valid objection to pull-based mathematics has been the uncomfortable notion of using a mathematical idea before knowing the techniques of its application. Not unreasonably, mathematicians regard the techniques as an essential part of what it means to understand an idea and how to apply it. Yet times, and technologies, change: sensitive and carefully-designed IT use can make it possible - perhaps even desirable - to use mathematical ideas before understanding the techniques.

The professional institutions, ICE and IStructE, and the JBM have had a policy of not being prescriptive about mathematics curriculum in assessing degree programmes for accreditation. Rather, each department is assessed on its own particular ways of teaching, and whether these lead to a satisfactory mathematical competence in its graduates.

Given the extent of the problems with mathematics, and the significant costs of innovation in curriculum and modes of delivery, there is a case perhaps for the institutions (together with cross-sector engineering organisations such as the Engineering Technology Board) to be more interventionist, both to push for change through the accreditation process, and actively to support curriculum change. In the boxes, we describe two examples of other professional institutions that have been doing this.

The experience with the Institute of Physics in particular suggests that there are great benefits to be gained from coordinated institutional response. Of course, physics differs from engineering in having a single professional institution; also it has taken the physics community about a decade of discussion to reach the present situation.

The advantage of a co-ordinated institutional response is that it unites the effort of individuals, particularly in the demanding area of developing IT-based curriculum, also since the institution has central concerns for the subject, it is possible to deploy IT in a way that is sympathetic for the subject, rather than the general 'top down' promotion of the use of IT which has been common in schools and universities over the past decade.

Comparison 1: The Institution of Electrical Engineers (IEE)

The IEE Degree Accreditation Committee convened a Curriculum Review Working Party in 2001, one of whose tasks was to investigate 'Mathematics in Schools', with the aim of having the IEE influence the key bodies in charge of school mathematics (DfES / QCA), in terms of standards of mathematics in schools and teacher training procedures, as well as the Engineering Council (UK), who guide the position of mathematics in engineering curricula through SARTOR. A report was delivered in summer 2002, with no action taken by the IEE up to now, although summary reports were published in a range of professional and engineering education journals.

'Mathematics demands careful thought, sustained attention and practice to an extent that students become fluent and at ease in their handling of formulae, equations and geometry. This does not come from group work or quickly. Targets need to be achieved younger than current fashion dictates: for example, 11-year-olds should be fluent in numerical fractions, so that 15-year-olds can be fluent in algebra, and 18-year-olds in calculus. The old 1950s syllabuses demonstrate that this is feasible: children are no less intelligent now. Applied Maths provided a very solid basis for Science / Engineering, with the concepts of vectors, forces, moments, rates of change, etc. Similarly, the approach of Euclidean geometry, with its emphasis on proof, provides the essential discipline for writing good software.' (Pyle 2002)

The IEE discussions appear to stand in some contrast to the JBM's; according to Professor Ian Pyle [personal communication], the software engineering industry is pressing for more mathematical content in degrees, not less. But, as in civil engineering, the key issue is the implicit mathematical understanding that contributes to the understanding of engineering principles. As we noted elsewhere, the IEE has been actively pursuing the development of problem-based learning in undergraduate degrees, and this is going to entail some shift in approaches to mathematics.

Comparison 2: The Institute of Physics (IoP)

The issues being faced by engineering now are rather similar to those faced in physics education, especially at A-level, where the decision was taken some years ago to untie the curriculum dependency with A-level mathematics. It seems to us that physics educators have debated and thought through the issues in ways that are of relevance to the current debate in undergraduate engineering, and also have gone a long way in developing and implementing new curriculum materials.

This has been in many ways an institutional response to an educational problem. The Institute of Physics (IoP) has a Post-16 Initiative [http://post16.iop.org] which has been coordinating the discussions and developments. To stimulate debate, a series of discussion booklets (downloadable from the IoP website) was produced, one of them dedicated to the relationships between physics and mathematics *(Carson 1999)*:

'Mathematical' understanding in physics [is] not simply the ability formally to manipulate algebraic equations, to substitute figures into formulae or to plot points on a graph, for example, but it is also the ability to understand the meaning encoded in the shape of a graph, to understand the qualitative relationships and connections between physical quantities and the ability to develop models that lead to real understanding of physical phenomena. …Much of the qualitative feel that we have for the subject is admittedly born of long experience with the quantitative mathematics but we should not shy away from developing the ability to reason qualitatively in our students…' (Carson 1999)

Working along these lines, the IoP has supported the development of a new course for A-level, Advancing Physics [http://advancingphysics.iop.org] with the integrated use of modelling software.

The current stage of the IoP initiative is looking at university physics courses. A major report was issued *(Institute of Physics 2001)* which recommended the appointment of a working group (currently active) to assess the development of new degree programme 'drawing heavily upon physics but being more interdisciplinary in focus … to build mathematical knowledge and competence during the course of study… [train] students for a broad base of technical or scientifically related employment areas … [for] young people expressing an interest in the physical sciences and engineering but finding themselves unable to fit into the conventional physics degree.' *(ibid, pp. 28-29)*.

4

Conclusions

1 We have found agreement from every quarter that undergraduate engineering students continue to need to know and to learn mathematics. The fundamental question is *what kind* of mathematics is needed and *when*.

2 The system of mathematical education in engineering formation is ripe for change-regulatory frameworks, entry routes to the profession, and post-14 school mathematics provision are all likely to experience major changes in the near future. Therefore there is a need to consider the *mathematical knowledge* that is required, by whom, and in what form. For example, geometry is a key area of knowledge for civil engineers that is currently under-taught in schools and universities, and there is reason to consider making geometry more of an organising theme for mathematics courses than is currently the norm.

3 There are possibilities in the 'new symbolisms' that practising engineers use, through software, to engage with mathematical ideas. These need to be critically examined in engineering education, alongside well-understood algebraic symbolism.

4 It is time to reconsider pedagogical approaches that can best 'deliver' the mathematical needs of students. Mathematics could benefit from being more 'pulled' into the context of design-oriented engineering teaching, rather than 'pushed' into students in the absence of a context. This entails a shift in approach from teaching mathematical techniques towards teaching through modelling and problem solving.

5 Carefully-designed IT use can make it possible to use mathematical ideas before *understanding* the techniques. In the pre-computational era, a strong objection to pull-based mathematics was that to use a mathematical idea properly required a detailed understanding of the techniques of its application. But times, and technologies, change.

6 There is a need for national leadership to stimulate, and to promote the spread of, the innovative work in curriculum design and delivery currently being carried out by enthusiastic individuals and individual departments. This is a role that the professional institutions (together with cross-sector engineering organisations such as the Engineering Technology Board) should be well placed to assume, as campaigners for the engineering profession and controllers of the accreditation mechanisms for engineering degrees.

5

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Glossary of abbreviations and organisations

ABET	Accreditation Board for Engineering and Technology (USA)	www.abet.org
EC(UK)	The Engineering Council (UK)	www.engc.org.uk
ETB	Engineering Technology Board	www.etechb.co.uk
EPC	The Engineering Professors Council	www.engprofc.ac.uk
HEFCE	Higher Education Funding Council for England	www.hefce.ac.uk
ICE	Institution of Civil Engineers	www.ice.org.uk
IEE	Institution of Electrical Engineers	www.iee.org.uk
IMA	Institute of Mathematics and its Applications	www.ima.org.uk
IoP	Institute of Physics	www.iop.org
IStructE	Institution of Structural Engineers	www.istructe.org.uk
JBM	Joint Board of Moderators	-
LTSN	Learning and Teaching Support Network	www.ltsn.ac.uk
	LTSN Centre for Engineering	www.ltsneng.ac.uk
	LTSN Maths, Stats and OR Network	ltsn.mathstore.ac.uk
QAA	Quality Assurance Agency for Higher Education	www.qaa.ac.uk
SARTOR	Standards and Routes to Registration	www.engc.org.uk/registration

Richard Noss is Professor of Mathematics Education, and Pro-director (ICT) of the Institute of Education, University of London. He has a Masters degree in pure mathematics, a PhD in mathematical education, and has taught mathematics at all levels of the education system. Professor Noss has directed and participated in more than 20 funded research studies of mathematical learning. He has edited and authored five books, including *Windows on Mathematical Meanings: Learning Cultures and Computers* co-authored with Celia Hoyles in 1996, and has published more than 100 journal articles and book chapters. Professor Noss is currently the editor-in-chief of the *International Journal of Computers for Mathematical Learning*. His research focuses on the ways mathematics is used in a variety of professions (including bankers, pilots, nurses) as well as by engineers (a recently-completed study, funded by the Economic and Social Research Council, is entitled *The Mathematical Components of Engineering Expertise*). He is also involved in the design of software that fosters the growth of mathematical knowledge in children and adults.

Phillip Kent is a Research Officer in the School of Mathematics, Science and Technology at the Institute of Education, University of London. He qualified initially as a physicist, and then gained a PhD in applied mathematics at Imperial College, London, where he subsequently taught and undertook curriculum development in undergraduate mathematics for eight years, with a particular focus on the integration of mathematical software into mathematics, science and engineering courses. One of the outcomes of this was a student textbook, *Experiments in Undergraduate Mathematics: A Mathematica-based Approach* (1996), which Phillip co-authored. He joined the Institute of Education in 2001, where he has carried out several research projects on the use of mathematics in the practices of technical workplaces. His research interests also encompass the use of mathematics by engineers and scientists, and the changes brought to the learning and application of mathematics (at all educational levels) by computer technology and mathematical software.

Mathematics in the University Education of Engineers

A Report to The Ove Arup Foundation

The Ove Arup Foundation was set up in 1989 to commemorate the life of Sir Ove Arup (1895-1988) as an educational trust related specifically to the built environment. The Foundation Trustees have commissioned two previous reports on the state of education for the construction industry. These were: 'Interdisciplinary skills for built environment professionals: A scoping study' by Prof David Gann and Dr Ammon Salter (1999), and 'Attracting the best and brightest: Broadening the appeal of engineering education' by Prof David Nethercot and Dr David Lloyd Smith (2001). There is currently much debate about what mathematical skills are needed for the engineers of tomorrow, and how and when these skills might best be acquired. In the light of this, the Foundation Trustees commissioned Professor Richard Noss and Doctor Phillip Kent to survey the current roles of mathematics in undergraduate engineering in the UK, with a particular focus on civil engineering, and to identify some visions of future directions for the teaching of mathematics.

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